Solutions

Each answer shows the steps, then the **result**.

2.1

- acceleration is **proportional to displacement** from a fixed point
- and **directed towards** that point (opposite to displacement)

2.2

- the **maximum displacement** from the equilibrium position

2.3

- $v_{\text{max}} = \omega x_0$

$$v_{\text{max}} = (5.0 \text{ rad s}^{-1})(0.040 \text{ m}) = \mathbf{0.20} \text{ m s}^{-1}$$

2.4

- they are **antiphase** (phase difference π / 180°)

3.1 B

- SHM: acceleration is **proportional to displacement and opposite in direction**

3.2

(a) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.50 \text{ s}} = \mathbf{12.6} \text{ rad s}^{-1}$$

(b) $v_{\text{max}} = \omega x_0 = (12.57 \text{ rad s}^{-1})(0.030 \text{ m}) = \mathbf{0.38} \text{ m s}^{-1}$

(c) $a_{\text{max}} = \omega^2 x_0$

$$a_{\text{max}} = (12.57 \text{ rad s}^{-1})^2(0.030 \text{ m}) = \mathbf{4.7} \text{ m s}^{-2}$$

3.3

- $v = \omega\sqrt{x_0^2 - x^2}$

$$v = (8.0 \text{ rad s}^{-1})\sqrt{(0.10 \text{ m})^2 - (0.060 \text{ m})^2} = (8.0 \text{ rad s}^{-1})\sqrt{0.0064 \text{ m}^2}$$

$$v = (8.0 \text{ rad s}^{-1})(0.080 \text{ m}) = \mathbf{0.64} \text{ m s}^{-1}$$

3.4

- displacement is a **sine** (or cosine) curve
- acceleration is the **same shape inverted** (negative)
- they are **antiphase** (180° out of phase)

3.5

(a) the line is **straight and through the origin**, so a is **proportional to** x

- its **negative gradient** shows a is in the **opposite direction** to x — i.e. $a = -\omega^2 x$, the SHM condition

(b) gradient $= -8.0 \text{ m s}^{-2}/0.05 \text{ m} = -160 \text{ s}^{-2} = -\omega^2$

$$\omega = \sqrt{160 \text{ s}^{-2}} = 12.6 \text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = \mathbf{0.50} \text{ s}$$

3.6

(a) a straight line through the origin shows $a \propto x$

- the negative gradient shows the acceleration is always directed **opposite** to the displacement, i.e. towards the equilibrium position — which is the definition of SHM

(b) $T = 6.4 \text{ s}/8 = 0.80 \text{ s}$

$$\omega = \frac{2\pi}{0.80 \text{ s}} = \mathbf{7.9} \text{ rad s}^{\mathbf{-1}}$$

(c) for SHM $a = -\omega^2 x$, so the magnitude of the gradient is ω^2

$$\omega = \sqrt{61 \text{ s}^{-2}} = 7.8 \text{ rad s}^{-1}$$

which agrees with (b)

(d) damping is the loss of energy from an oscillating system to its surroundings, usually through **resistive forces**

- so the **amplitude decreases** with time

(e) a curve starting at $+x_0$

- oscillating with the **same period** T so that it still completes two cycles by $2T$
- with each maximum displacement **smaller** than the one before, decaying smoothly