

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **acceleration is proportional to the displacement** from a fixed point, and is always directed **towards** that point
- $a = -\omega^2 x$

2.2

- the restoring force from the spring is $F = -kx$
- apply Newton's second law

$$a = \frac{F}{m} = -\frac{k}{m}x$$

- comparing with $a = -\omega^2 x$ gives $\omega^2 = \frac{k}{m}$, so

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

2.3

- the restoring force is $-mg \sin \theta$, which is proportional to $\sin \theta$ and **not** to the displacement itself
- only for **small angles** does $\sin \theta \approx \theta$, making the force proportional to x ; for a large amplitude the approximation fails and the motion is no longer simple harmonic

2.4

- find ω from the period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = 7.85 \text{ rad s}^{-1}$$

- then $a_0 = \omega^2 x_0$, with the amplitude in **metres**

$$a_0 = (7.85 \text{ rad s}^{-1})^2 \times 0.024 \text{ m} = \mathbf{1.5 \text{ m s}^{-2}}$$

2.5

- the lag is a fraction $\frac{\Delta t}{T} = \frac{0.15 \text{ s}}{0.60 \text{ s}} = \frac{1}{4}$ of a cycle

$$\varphi = 2\pi \times \frac{1}{4} = \frac{\pi}{2} \text{ rad (1.6 rad)}$$

2.6

- displacement and velocity: $\frac{\pi}{2}$ rad, with the velocity **ahead**
- displacement and acceleration: π rad – exactly out of phase

3.1 C

- $a = -\omega^2 x$, so the gradient of the line is $-\omega^2$, which is negative
- therefore $\omega = \sqrt{-\text{gradient}}$. **A** and **B** forget the square root; **D** gives the square root of a negative number

3.2

(a)

- at equilibrium the upthrust already balances the weight, so the resultant force comes only from the **extra** upthrust
- the extra volume submerged is Ax , so the extra upthrust is ρAxg , acting **upwards** while x is measured **downwards**

$$F = -\rho Agx$$

(b)

- apply Newton's second law

$$a = \frac{F}{m} = -\left(\frac{\rho Ag}{m}\right)x$$

- ρ , A , g and m are constant, so $a \propto -x$: the motion is simple harmonic, with

$$\omega^2 = \frac{\rho Ag}{m}$$

(c)

- substitute, keeping units

$$\omega^2 = \frac{1000 \text{ kg m}^{-3} \times 2.8 \times 10^{-3} \text{ m}^2 \times 9.81 \text{ m s}^{-2}}{0.45 \text{ kg}} = 61.0 \text{ s}^{-2}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{7.81 \text{ rad s}^{-1}} = \mathbf{0.80 \text{ s}}$$

3.3

(a)

- for a small angle the restoring force is $-mg \frac{x}{L}$, so $a = -\frac{g}{L}x$ and $\omega^2 = \frac{g}{L}$
- therefore

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \implies T^2 = \frac{4\pi^2}{g}L$$

(b)

- the gradient is $\frac{4\pi^2}{g}$, so rearrange for g

$$g = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{4.06 \text{ s}^2 \text{ m}^{-1}} = \mathbf{9.7 \text{ m s}^{-2}}$$

(c)

- any **two** of: the angle of swing is **small**, so that $\sin \theta \approx \theta$; the mass of the **string is negligible**; the bob behaves as a **point mass**; **air resistance** and other damping are negligible; the string does not stretch

(d)

- find the period from the graph, then ω

$$T^2 = 4.06 \text{ s}^2 \text{ m}^{-1} \times 0.65 \text{ m} = 2.64 \text{ s}^2, \quad T = 1.62 \text{ s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.62 \text{ s}} = 3.87 \text{ rad s}^{-1}$$

- then $v_0 = \omega x_0$, with the amplitude in metres

$$v_0 = 3.87 \text{ rad s}^{-1} \times 0.035 \text{ m} = \mathbf{0.14 \text{ m s}^{-1}}$$

(e)

- use $v = \omega \sqrt{x_0^2 - x^2}$

$$v = 3.87 \text{ rad s}^{-1} \times \sqrt{(0.035 \text{ m})^2 - (0.020 \text{ m})^2} = 3.87 \times 0.0287 \text{ m} = \mathbf{0.11 \text{ m s}^{-1}}$$

(f)

- no change**: the mass cancelled in $a = -\frac{g}{L}x$, so the period depends only on L and g