Solutions

Each answer shows the steps, then the **result**.

2.1

- the **acceleration is proportional to the displacement** from a fixed point, and is always directed **towards** that point
- $a = -\omega^2 x$

2.2

- the restoring force from the spring is $F = -kx$
- apply Newton's second law

$$a = \frac{F}{m} = -\frac{k}{m}x$$

- comparing with $a = -\omega^2 x$ gives $\omega^2 = \frac{k}{m}$, so

$$T = \frac{2\pi}{\omega} = \mathbf{2\pi\sqrt{\frac{m}{k}}}$$

2.3

- the restoring force is $-mg\sin\theta$, which is proportional to $\sin\theta$ and **not** to the displacement itself
- only for **small angles** does $\sin\theta \approx \theta$, making the force proportional to x; for a large amplitude the approximation fails and the motion is no longer simple harmonic

2.4

- find ω from the period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80\ \text{s}} = 7.85\ \text{rad s}^{-1}$$

- then $a_0 = \omega^2 x_0$, with the amplitude in **metres**

$$a_0 = (7.85\ \text{rad s}^{-1})^2 \times 0.024\ \text{m} = \mathbf{1.5}\ \text{m s}^{\mathbf{-2}}$$

2.5

- the lag is a fraction $\frac{\Delta t}{T} = \frac{0.15\ \text{s}}{0.60\ \text{s}} = \frac{1}{4}$ of a cycle

$$\varphi = 2\pi \times \frac{1}{4} = \frac{\pi}{\mathbf{2}}\ \text{rad (1.6 rad)}$$

2.6

- displacement and velocity: $\frac{\pi}{2}$ rad, with the velocity **ahead**
- displacement and acceleration: π rad – exactly out of phase

3.1 C

- $a = -\omega^2 x$, so the gradient of the line is $-\omega^2$, which is negative
- therefore $\omega = \sqrt{-\text{gradient}}$. **A** and **B** forget the square root; **D** gives the square root of a negative number

3.2

(a)

- at equilibrium the upthrust already balances the weight, so the resultant force comes only from the **extra** upthrust
- the extra volume submerged is Ax, so the extra upthrust is ρAxg, acting **upwards** while x is measured **downwards**

$$F = -\rho Agx$$

(b)

- apply Newton's second law

$$a = \frac{F}{m} = -\left(\frac{\rho Ag}{m}\right)x$$

- ρ, A, g and m are constant, so $a \propto -x$: the motion is simple harmonic, with

$$\omega^2 = \frac{\rho Ag}{m}$$

(c)

- substitute, keeping units

$$\omega^2 = \frac{1000\ \text{kg m}^{-3} \times 2.8 \times 10^{-3}\ \text{m}^2 \times 9.81\ \text{m s}^{-2}}{0.45\ \text{kg}} = 61.0\ \text{s}^{-2}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{7.81\ \text{rad s}^{-1}} = \mathbf{0.80}\ \text{s}$$

3.3

(a)

- for a small angle the restoring force is $-mg\dfrac{x}{L}$, so $a = -\dfrac{g}{L}x$ and $\omega^2 = \dfrac{g}{L}$
- therefore

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \implies T^2 = \frac{4\pi^2}{g}L$$

(b)

- the gradient is $\dfrac{4\pi^2}{g}$, so rearrange for g

$$g = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{4.06\ \text{s}^2\ \text{m}^{-1}} = \mathbf{9.7}\ \text{m s}^{-2}$$

(c)

- any **two** of: the angle of swing is **small**, so that $\sin\theta \approx \theta$; the mass of the **string is negligible**; the bob behaves as a **point mass**; **air resistance** and other damping are negligible; the string does not stretch

(d)

- find the period from the graph, then ω

$$T^2 = 4.06\ \text{s}^2\ \text{m}^{-1} \times 0.65\ \text{m} = 2.64\ \text{s}^2, \qquad T = 1.62\ \text{s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.62\ \text{s}} = 3.87\ \text{rad s}^{-1}$$

- then $v_0 = \omega x_0$, with the amplitude in metres

$$v_0 = 3.87\ \text{rad s}^{-1} \times 0.035\ \text{m} = \mathbf{0.14}\ \text{m s}^{-1}$$

(e)

- use $v = \omega\sqrt{x_0^2 - x^2}$

$$v = 3.87\ \text{rad s}^{-1} \times \sqrt{(0.035\ \text{m})^2 - (0.020\ \text{m})^2} = 3.87 \times 0.0287\ \text{m} = \mathbf{0.11}\ \text{m s}^{-1}$$

(f)

- **no change**: the mass cancelled in $a = -\dfrac{g}{L}x$, so the period depends only on L and g