Solutions

Each answer shows the steps, then the **result**.

2.1

- any **three** of: a large number of molecules in **random** motion; the **volume of the molecules** is negligible compared with the volume of the container; collisions are **perfectly elastic**; the duration of a collision is negligible compared with the time between collisions; there are **no forces** between molecules except during collisions

2.2

- momentum is a **vector**: the molecule arrives with $+mv_x$ and leaves with $-mv_x$
- the **change** is $(-mv_x) - (+mv_x) = -2mv_x$, so its magnitude is $2mv_x$

2.3

- after rebounding, the molecule must cross the box and return before it can hit the same wall again, a distance of $2L$
- its component of velocity in that direction is v_x and is unchanged in magnitude, so

$$\Delta t = \frac{\text{distance}}{\text{speed}} = \frac{2L}{v_x}$$

2.4

- the **mean-square speed** $\langle c^2\rangle$ is the average of the squares of the speeds of all the molecules
- the **root-mean-square speed** is the square root of that, $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$

2.5

- the motion is **random**, so no direction is preferred and $\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle$
- since $c^2 = v_x^2 + v_y^2 + v_z^2$, taking means gives $\langle c^2\rangle = 3\langle v_x^2\rangle$, so $\langle v_x^2\rangle = \frac{1}{3}\langle c^2\rangle$

3.1 A

- $c_{\text{r.m.s.}} = \sqrt{\dfrac{3kT}{m}}$, so $c_{\text{r.m.s.}} \propto \sqrt{T}$
- doubling T multiplies the speed by $\sqrt{2}$. **B** is the standard error – it is the mean *kinetic energy* that doubles, not the speed

3.2

(a)

- both expressions equal pV, so set them equal

$$\tfrac{1}{3}Nm\langle c^2\rangle = NkT$$

- the N cancels

$$\tfrac{1}{3}m\langle c^2\rangle = kT$$

- multiply both sides by $\frac{3}{2}$

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

(b)

- convert the temperature to kelvin, $T = 300$ K

$$E = \tfrac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 300\ \text{K} = \mathbf{6.2 \times 10^{-21}}\ \text{J}$$

(c)

- thermodynamic temperature is a measure of the **mean translational kinetic energy** of the molecules, and it is the same for every ideal gas at that temperature

3.3

(a)

- one mole contains N_A atoms

$$m = \frac{4.0 \times 10^{-3}\ \text{kg mol}^{-1}}{6.02 \times 10^{23}\ \text{mol}^{-1}} = \mathbf{6.6 \times 10^{-27}}\ \text{kg}$$

(b)

- rearrange $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$

$$\langle c^2\rangle = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 300\ \text{K}}{6.6 \times 10^{-27}\ \text{kg}} = 1.87 \times 10^6\ \text{m}^2\ \text{s}^{-2}$$

$$c_{\text{r.m.s.}} = \sqrt{1.87 \times 10^6\ \text{m}^2\ \text{s}^{-2}} = \mathbf{1.4 \times 10^3}\ \text{m s}^{-1}$$

(c)

- at the same temperature both gases have the **same mean translational kinetic energy**, $\frac{3}{2}kT$
- so $\frac{1}{2}m\langle c^2\rangle$ is the same for both, and $c_{\text{r.m.s.}} \propto \dfrac{1}{\sqrt{m}}$
- an oxygen molecule has **eight times** the mass, so its r.m.s. speed is $\dfrac{1}{\sqrt{8}}$ of the helium value – about **2.8 times smaller**, roughly 480 m s^{-1}

(d)

- the r.m.s. speed **increases by a factor of** $\sqrt{2}$, to about 1.9×10^3 m s^{-1}
- because $c_{\text{r.m.s.}} \propto \sqrt{T}$, and the thermodynamic temperature has doubled

(e)

- heating raises the mean translational kinetic energy, so the molecules move **faster** on average
- they therefore hit the walls **more frequently**
- and each collision produces a **greater change of momentum**; both effects raise the average force on the walls, and with the area unchanged the pressure rises