

15.2 Equation of state

Name: _____ Class: _____ Date: _____

Total: 23 marks

KEY WORDS ideal gas 理想气体 • boltzmann constant 玻尔兹曼常量
 • equation of state 状态方程 • molar gas constant 摩尔气体常量 • pressure 压强

Objective

Build the skills to answer exam questions on the **equation of state** of an ideal gas.

You must be able to:

- use $pV = nRT$ and $pV = NkT$ (with T in kelvin)
- recall R , k and that $k = R/N_A$
- read a pV - T **graph**: gradient $= Nk$, so $N = \text{gradient}/k$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Equation of state

An **ideal gas** obeys $pV \propto T$. Two equal forms:

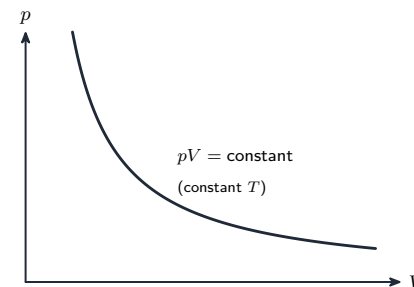
$$pV = nRT \quad (n = \text{moles}), \quad pV = NkT \quad (N = \text{molecules}),$$

with $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$, and $k = R/N_A$. **Always use Pa, m³, K.**

Worked example. 0.50 mol at 300 K in 0.020 m³: $p = \frac{nRT}{V} = \frac{0.50 \times 8.31 \times 300}{0.020} = 6.2 \times 10^4 \text{ Pa}$.

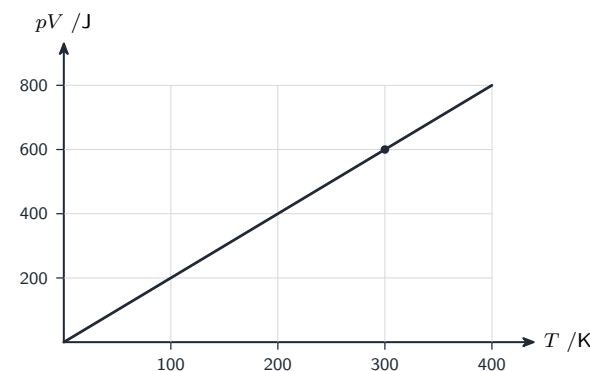
■ Boyle's law

At constant T (and amount), $p \propto 1/V$, so $pV = \text{constant}$:



■ Reading a pV - T graph

For a fixed amount of gas, $pV = NkT$, so a graph of pV against T (in kelvin) is a **straight line through the origin with gradient Nk** :



A straight line **through the origin** confirms the gas is **ideal** ($pV \propto T$). To find the number of molecules, read the **gradient** and divide by k : $N = \frac{\text{gradient}}{k}$.

Worked example. The line above passes through (300 K, 600 J), so gradient $= 600/300 = 2.0 \text{ J K}^{-1} = Nk$; thus $N = \frac{2.0}{1.38 \times 10^{-23}} = 1.4 \times 10^{23}$.

2 Practice

Now apply the methods above.

2.1 State what is meant by an **ideal gas**. [1]

2.2 Write the **two** forms of the ideal-gas equation of state. [2]

2.3 State the value of the **molar gas constant** R . [1]

2.4 A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to. [1]

3 Exam-style questions

3.1 In $pV = nRT$, the temperature T must be in: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A °C
B K
C J
D mol

3.2 0.50 mol of an ideal gas at 300 K occupies 0.020 m³. Calculate the **pressure**. ($R = 8.31$.) [3]

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through (300 K, 600 J).

(a) State what this tells you about the gas. [1]

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 State **Boyle's law**. [1]

3.5 For an ideal gas, two equations are used together:

$$pV = NkT \quad \text{and} \quad pV = \frac{1}{3}Nm\langle c^2 \rangle$$

(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k . [2]

(b) **State** the meanings of the symbols m and $\langle c^2 \rangle$ in the second equation. [2]

(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas. [3]

(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**. [2]
