Solutions

Each answer shows the steps, then the **result**.

2.1

- $g = GM/r^2$

2.2

- $g \propto 1/r^2$ (inverse-square)

2.3

- at $2R$, g is $1/2^2 = 1/4$ of the surface value

$$g = \frac{9.8 \text{ N kg}^{-1}}{4} = \mathbf{2.45} \text{ N kg}^{-1}$$

2.4

- a small height change is **tiny compared with the Earth's radius**, so r (and hence $g \propto 1/r^2$) barely changes

3.1 C

- $g \propto 1/r^2$, so at $2R$ the field is **one quarter**

3.2

- $g = GM/R^2$

$$g = \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(7.3 \times 10^{22} \text{ kg})}{(1.7 \times 10^6 \text{ m})^2} = \mathbf{1.7} \text{ N kg}^{-1}$$

3.3

- force on a test mass m: $F = GMm/r^2$
- field strength $g = F/m = GM/r^2$

3.4

- $g \propto 1/R^2$, so 1/4 of Earth's

3.5

- a curve **starting at** g_0 at $r = R$ and **falling** as an inverse square
- to $g_0/4$ at $2R$, $g_0/9$ at $3R$, $g_0/16$ at $4R$ — approaching (but never reaching) zero

3.6

(a) outside the sphere all its mass may be treated as concentrated at the centre, giving $g \propto 1/x^2$; inside, only the mass **closer to the centre** than the point contributes, so the field falls towards zero at the centre instead of rising

(b) the gravitational force provides the centripetal force: $GMm/r^2 = mv^2/r$

- the satellite mass m appears on both sides and **cancels**
- $v^2 = GM/r$, so $v = \sqrt{GM/r}$

(c) $T = 2\pi r/v = 2\pi r\sqrt{r/(GM)}$

- $T^2 = 4\pi^2 r^3/(GM)$
- that is $T^2 \propto r^3$, **Kepler's third law**

(d) $T = 24 \times 3600\ \text{s} = 86\,400\ \text{s}$

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{(4.00 \times 10^{14}\ \text{m}^3\,\text{s}^{-2})(86\,400\ \text{s})^2}{4\pi^2} = 7.56 \times 10^{22}\ \text{m}^3$$

- $r = \mathbf{4.2 \times 10^7}$ m

(e) it must orbit **above the equator**, in the plane of the equator

- and travel **west to east**, the same direction as the Earth's rotation