

Solutions

Each answer shows the steps, then the **result**.

2.1

$$\bullet \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3}{6.0} + \frac{2}{6.0} + \frac{1}{6.0} = 1.0 \Rightarrow R = \mathbf{1.0 \Omega}$$

- the answer is smaller than the smallest resistor, as it must be

2.2

- the parallel pair is now two equal resistors, so $R_p = \frac{6.0}{2} = 3.0 \Omega$
- in series with the others: $R_{XY} = 5.0 + 3.0 + 3.0 = \mathbf{11 \Omega}$

2.3

- the second resistor gives the charge an extra path, so for the same p.d. the total current is larger
- $R = V/I$, so a larger current for the same p.d. means a smaller resistance

2.4

- in series the current is the same in both, and $P = I^2 R$
- the $\mathbf{8.0 \Omega}$ resistor, dissipating **twice** the power of the 4.0Ω

2.5

- in parallel the p.d. is the same across both, and $P = V^2/R$, so the $\mathbf{4.0 \Omega}$ resistor dissipates the greater power
- the answer changes because the resistors now share the p.d. instead of the current

3.1 C

- parallel pair: $\frac{1}{R_p} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{2.0}$, so $R_p = 2.0 \Omega$
- total = $4.0 + 2.0 = 6.0 \Omega$, so $I = \frac{12}{6.0} = 2.0 \text{ A}$

3.2

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$
- substitute $I = V/R$ for each: $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$, so $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

3.3

$$(a) \frac{1}{R_p} = \frac{1}{9.0} + \frac{1}{18} = \frac{2}{18} + \frac{1}{18} = \frac{3}{18}$$

$$\bullet R_p = \frac{18}{3} = \mathbf{6.0 \Omega}$$

$$(b) \text{ total resistance} = 6.0 + 6.0 = 12 \Omega$$

$$I = \frac{18}{12} = \mathbf{1.5 \text{ A}}$$

$$(c) V_p = I R_p = (1.5)(6.0) = \mathbf{9.0 \text{ V}}$$

- $I_9 = \frac{9.0}{9.0} = \mathbf{1.0 \text{ A}}$ and $I_{18} = \frac{9.0}{18} = \mathbf{0.50 \text{ A}}$
- check: $1.0 + 0.50 = 1.5 \text{ A}$, the current in the battery

$$(d) \text{ the } \mathbf{6.0 \Omega} \text{ resistor}$$

- it carries the whole 1.5 A while each branch carries only part of it, and $P = I^2 R$: 13.5 W against 9.0 W and 4.5 W

3.4

$$(a) I_1 = I_2 + I_3$$

$$(b) \text{ parallel pair: } \frac{1}{R_p} = \frac{1}{12} + \frac{1}{4.0} = \frac{1}{12} + \frac{3}{12} = \frac{4}{12}, \text{ so } R_p = 3.0 \Omega$$

- total = $3.0 + 3.0 = 6.0 \Omega$, so $I_1 = \frac{12}{6.0} = \mathbf{2.0 \text{ A}}$
- p.d. across the pair = $(2.0)(3.0) = 6.0 \text{ V}$
- $I_2 = \frac{6.0}{12} = \mathbf{0.50 \text{ A}}$ and $I_3 = \frac{6.0}{4.0} = \mathbf{1.5 \text{ A}}$

$$(c) V_1 = I_1 R_1$$

$$V_1 = (2.0)(3.0) = \mathbf{6.0 \text{ V}}$$

$$(d) P = I^2 R \text{ for each resistor}$$

- $P_1 = (2.0)^2(3.0) = 12 \text{ W}$, $P_2 = (0.50)^2(12) = 3.0 \text{ W}$, $P_3 = (1.5)^2(4.0) = 9.0 \text{ W}$, total = $\mathbf{24 \text{ W}}$
- the cell supplies $\varepsilon I_1 = (12)(2.0) = 24 \text{ W}$, the same, illustrating **conservation of energy** – which is what Kirchhoff's second law expresses