Solutions

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3}{6.0} + \frac{2}{6.0} + \frac{1}{6.0} = 1.0 \Rightarrow R = \mathbf{1.0}\ \Omega$$

- the answer is smaller than the smallest resistor, as it must be

2.2

- the parallel pair is now two equal resistors, so $R_\mathrm{p} = \dfrac{6.0}{2} = 3.0\ \Omega$
- in series with the others: $R_{XY} = 5.0 + 3.0 + 3.0 = \mathbf{11}\ \Omega$

2.3

- the second resistor gives the charge an extra path, so for the same p.d. the total current is larger
- $R = V/I$, so a larger current for the same p.d. means a smaller resistance

2.4

- in series the current is the same in both, and $P = I^2R$
- the **8.0** Ω resistor, dissipating **twice** the power of the 4.0 Ω

2.5

- in parallel the p.d. is the same across both, and $P = V^2/R$, so the **4.0** Ω resistor dissipates the greater power
- the answer changes because the resistors now share the p.d. instead of the current

3.1 C

- parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = \dfrac{1}{2.0}$, so $R_\mathrm{p} = 2.0\ \Omega$
- total $= 4.0 + 2.0 = 6.0\ \Omega$, so $I = \dfrac{12}{6.0} = 2.0$ A

3.2

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$
- substitute $I = V/R$ for each: $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$, so $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1R_2} \Rightarrow R = \frac{R_1R_2}{R_1 + R_2}$$

3.3

(a) $\dfrac{1}{R_{\text{p}}} = \dfrac{1}{9.0} + \dfrac{1}{18} = \dfrac{2}{18} + \dfrac{1}{18} = \dfrac{3}{18}$

- $R_{\text{p}} = \dfrac{18}{3} = \mathbf{6.0}\ \Omega$

(b) total resistance $= 6.0 + 6.0 = 12\ \Omega$

$$I = \frac{18}{12} = \mathbf{1.5}\ \text{A}$$

(c) $V_{\text{p}} = IR_{\text{p}} = (1.5)(6.0) = \mathbf{9.0}\ \text{V}$

- $I_9 = \dfrac{9.0}{9.0} = \mathbf{1.0}$ A and $I_{18} = \dfrac{9.0}{18} = \mathbf{0.50}$ A
- check: $1.0 + 0.50 = 1.5$ A, the current in the battery

(d) the **6.0** Ω resistor

- it carries the whole 1.5 A while each branch carries only part of it, and $P = I^2R$: 13.5 W against 9.0 W and 4.5 W

3.4

(a) $I_1 = I_2 + I_3$

(b) parallel pair: $\dfrac{1}{R_{\text{p}}} = \dfrac{1}{12} + \dfrac{1}{4.0} = \dfrac{1}{12} + \dfrac{3}{12} = \dfrac{4}{12}$, so $R_{\text{p}} = 3.0\ \Omega$

- total $= 3.0 + 3.0 = 6.0\ \Omega$, so $I_1 = \dfrac{12}{6.0} = \mathbf{2.0}$ A
- p.d. across the pair $= (2.0)(3.0) = 6.0$ V
- $I_2 = \dfrac{6.0}{12} = \mathbf{0.50}$ A and $I_3 = \dfrac{6.0}{4.0} = \mathbf{1.5}$ A

(c) $V_1 = I_1R_1$

$$V_1 = (2.0)(3.0) = \mathbf{6.0}\ \text{V}$$

(d) $P = I^2R$ for each resistor

- $P_1 = (2.0)^2(3.0) = 12$ W, $P_2 = (0.50)^2(12) = 3.0$ W, $P_3 = (1.5)^2(4.0) = 9.0$ W, total $=$ **24** W
- the cell supplies $\varepsilon I_1 = (12)(2.0) = 24$ W, the same, illustrating **conservation of energy** – which is what Kirchhoff's second law expresses