A-LEVEL PHYSICS • EXERCISE SHEET

1.4 Scalars and vectors

Name: ____________ Class: ________ Date: __________ **Total: 34 marks**

KEY WORDS vector 矢量 • scalar 标量 • component 分量 • perpendicular 垂直 • magnitude 大小

Objective

Build the skills to answer the real exam questions on **scalars and vectors**. In the exam, 1.4 is tested by **multiple-choice** questions and by short parts inside longer questions: classify a quantity, find a **resultant**, or **resolve** a vector into components.

You must be able to:

- tell a **scalar 标量** from a **vector 矢量** and complete a classification table
- find the **resultant 合矢量** of two **perpendicular 垂直** vectors (size and direction)
- give the direction of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$
- find the resultant of **two equal forces** at an angle
- **resolve 分解** any vector (a force *or* a velocity) into two perpendicular **components 分量**
- add three or more **coplanar 共面** vectors by components
- **sketch** each component of a projectile's velocity against time

1 Worked examples

Study these first. Each one shows the method for an exam question type used below — read them and you can do the Practice and Exam-style questions yourself.

■ A — Scalar or vector?

A scalar has size only; a vector has size **and** direction. **Quick test:** if "in which direction?" makes sense, it is a vector. In the exam you often tick a table:

Quantity	Scalar	Vector
speed	✓	
acceleration		✓
energy	✓	
momentum		✓

(Common scalars: mass, time, temperature, energy, distance, speed. Common vectors: displacement, velocity, acceleration, force, momentum.)

■ B — Add two perpendicular vectors

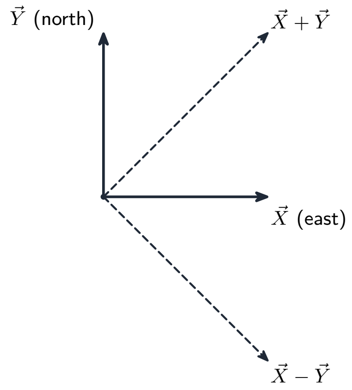
Draw them **tip to tail**; the resultant closes the triangle. For two vectors at 90° the size is $|\vec{R}| = \sqrt{X^2+Y^2}$ and the direction is $\tan\theta = Y/X$.

Worked example. A swimmer heads north at 1.2 m s^{-1}; the river flows east at 0.5 m s^{-1}:

$$|\vec{R}| = \sqrt{1.2^2+0.5^2} = 1.3 \text{ m s}^{-1}, \qquad \tan\theta = \tfrac{0.5}{1.2} \Rightarrow \theta \approx 23^\circ \text{ east of north.}$$

■ C — Direction of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$

To **add**, place the arrows tip to tail. To **subtract**, reverse the second arrow first: $\vec{X}-\vec{Y} = \vec{X}+(-\vec{Y})$, where $-\vec{Y}$ is the same size as $\vec{Y}$ but points the opposite way.



With $\vec{X}$ east and $\vec{Y}$ north: $\vec{X}+\vec{Y}$ points **north-east**, and $\vec{X}-\vec{Y}$ points **south-east**.

■ C2 — Adding three or more coplanar vectors

Coplanar 共面 vectors all lie in the same flat plane — every vector in this course is coplanar. For three or more, drawing triangles gets messy, so **use components**: add all the x-parts, add all the y-parts, then recombine.

Worked example. Three coplanar forces act at a point: 2.0 N east, 3.0 N north, 4.0 N west.

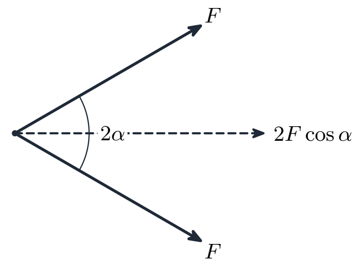
$$\sum x = 2.0 - 4.0 = -2.0 \text{ N}, \qquad \sum y = +3.0 \text{ N}$$

$$R = \sqrt{2.0^2+3.0^2} = 3.6 \text{ N}, \qquad \tan\theta = \tfrac{3.0}{2.0} \Rightarrow \theta = 56^\circ \text{ north of west}$$

Take one direction as positive and stay with it — a lost minus sign is the usual error.

■ **D — Two equal forces at an angle**

Two equal forces of size F with an angle 2α between them have a resultant of size $2F\cos\alpha$, along the line that **cuts the angle in half**.

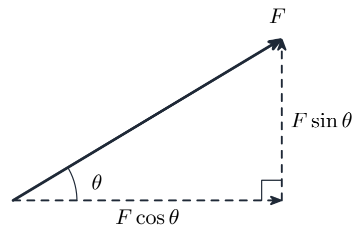


Worked example. Two 6.0 N forces act with 60° between them. Here $2\alpha = 60°$, so $\alpha = 30°$:

$$R = 2F\cos\alpha = 2(6.0)\cos 30° = 10.4 \text{ N}.$$

■ **E — Resolve a vector into components**

Any vector $\vec{v}$ at angle θ to the horizontal splits into a horizontal part $v\cos\theta$ and a vertical part $v\sin\theta$ — this works for a force **or** a velocity 速度.



Force example. A 50 N force at 30°: horizontal = $50\cos 30° = 43$ N, vertical = $50\sin 30° = 25$ N.

Velocity example. A ball kicked at 28 m s^{-1}, 34° above the horizontal: $v_H = 28\cos 34° = 23$ m s^{-1}, $v_V = 28\sin 34° = 16$ m s^{-1}.

On a slope of angle θ, the weight splits into $W\sin\theta$ **along** the slope and $W\cos\theta$ **into** the slope.

■ **F — The two components of a projectile, against time**

Once the components are found, each one has its own simple graph — and the exam asks you to sketch both on one grid:

- **horizontal** v_H: a **horizontal straight line** (no horizontal force, so it never changes).
- **vertical** v_V: a **straight line sloping down**, starting at $+v\sin\theta$, crossing zero at the top of the flight, and reaching $-v\sin\theta$ on landing (taking up as positive).

The gradient of that second line is $-g$, and the time to the top is v_V/g. Label each line, and use the whole time axis you are given.

2 Practice

Now apply the methods above.

2.1 Complete the table by ticking **scalar** or **vector** for each quantity. [3]

Quantity	Scalar	Vector
distance		
force		
temperature		
velocity		
power		
acceleration		

2.2 Forces of 3.0 N east and 4.0 N north act at a point. Find the **magnitude** 大小 of the resultant. [2]

2.3 Resolve a 50 N force acting at 30° to the horizontal into its horizontal and vertical components. [2]

2.4 A ball leaves the ground at 20 m s^{-1} at 60° above the horizontal. Find the **horizontal** and **vertical** components of its velocity. [2]

2.5 Two forces, each 5.0 N, act at a point with 90° between them. Find the **magnitude** of the resultant. [2]

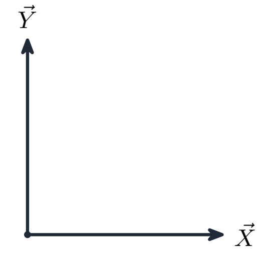
2.6 Three **coplanar** forces act at a point: 6.0 N east, 2.0 N west and 5.0 N north. Find the **magnitude** and the **direction** of the resultant. [3]

3 Exam-style questions

3.1 Which of these is a **vector** quantity? [1]

A energy
B power
C displacement
D temperature

3.2 Two vectors $\vec{X}$ and $\vec{Y}$ are shown: $\vec{X}$ points east and $\vec{Y}$ points north. What are the directions of $\vec{X}+\vec{Y}$ and $\vec{X}-\vec{Y}$? [1]



A $\vec{X}+\vec{Y}$ north-east; $\vec{X}-\vec{Y}$ south-east
B $\vec{X}+\vec{Y}$ north-east; $\vec{X}-\vec{Y}$ north-west
C $\vec{X}+\vec{Y}$ south-west; $\vec{X}-\vec{Y}$ south-east
D $\vec{X}+\vec{Y}$ south-east; $\vec{X}-\vec{Y}$ north-east

3.3 Two forces of 6.0 N and 8.0 N act at a point at right angles to each other. What is the magnitude of the resultant force? [1]

A 2.0 N
B 10 N
C 14 N
D 48 N

3.4 A stone is thrown with a velocity of 15 m s^{-1} at 40° above the horizontal.

(a) Find the **horizontal** and **vertical** components of this velocity. [2]

(b) Air resistance is negligible. State which component stays **constant** during the flight, and why. [2]

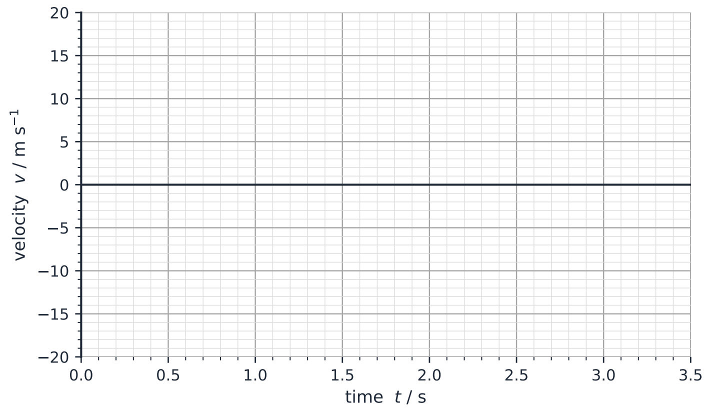
3.5 Two forces, each 7.0 N, act at a point with 80° between them. Find the **magnitude** of the resultant. [2]

3.6 A player kicks a ball from level ground with a velocity of 21 m s^{-1} at 55° to the horizontal. Air resistance is negligible.

(a) **Calculate** the horizontal component v_H and the vertical component v_V of the velocity of the ball as it leaves the ground. [2]

(b) **Show that** the ball reaches its maximum height at time $t = 1.75$ s. [1]

(c) The ball lands at time $t = 3.5$ s. On the grid below, **sketch** the variation with time t of v_H and of v_V, from $t = 0$ to $t = 3.5$ s. Take the upward direction as positive. **Label** your lines H and V. [4]



(d) **Determine** the magnitude and the direction of the velocity of the ball just before it lands. [3]

(e) State why the horizontal component of the velocity does not change during the flight. [1]