

1.3 Errors and uncertainties

Name: _____ Class: _____ Date: _____

Total: 36 marks

KEY WORDS uncertainty 不确定度 • percentage uncertainty 百分比不确定度
• absolute uncertainty 绝对不确定度 • accuracy 准确度 • systematic error 系统误差

Objective

Build the skills to answer exam questions on **errors and uncertainties** — error types, precision vs accuracy, and combining uncertainties.

You must be able to:

- explain **systematic** (incl. **zero**) and **random** errors
- distinguish **precision** 精密度 from **accuracy** 准确度
- combine **absolute** and **percentage** uncertainties in a derived quantity
- **justify** the number of significant figures in a calculated result
- decide, from the uncertainties, whether two results **agree**

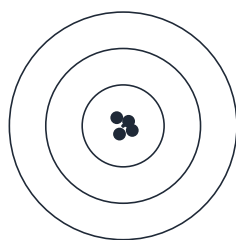
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

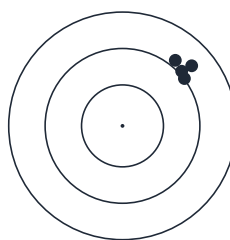
■ Errors, precision and accuracy

- A **systematic error** shifts every reading the **same way** (a **zero error**, a wrong calibration). Repeating does not reveal it; it harms **accuracy**.
- A **random error** scatters readings **both ways**. Taking the **mean** of many repeats reduces it; it harms **precision**.

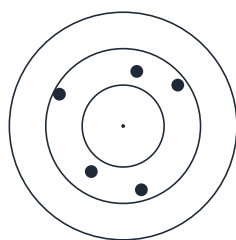
Precision = how close repeats are to each other; **accuracy** = how close they are to the true value:



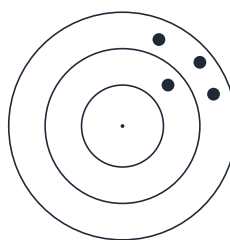
precise
and accurate



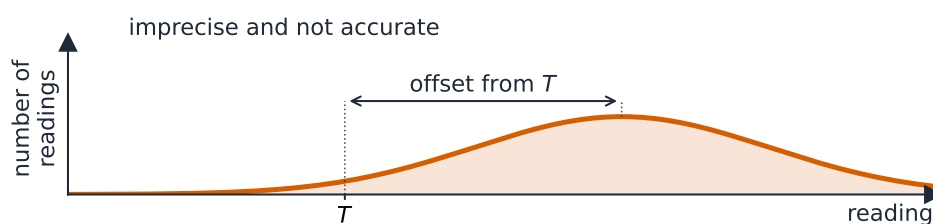
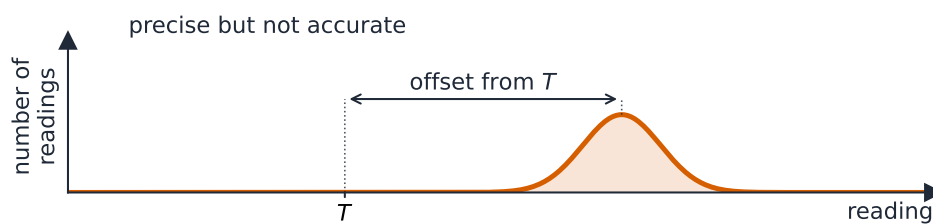
precise but
not accurate



accurate but
not precise



neither



accuracy: how close readings are to the true value (systematic error reduces accuracy)

Precision contrasted with accuracy

■ Uncertainty in one quantity

Write a measurement as $x \pm \Delta x$. The **percentage uncertainty** is $\frac{\Delta x}{|x|} \times 100\%$. E.g.

$$(2.50 \pm 0.05) \text{ m} \rightarrow \frac{0.05}{2.50} \times 100 = 2\%.$$

■ Combining uncertainties

- **Add / subtract** → add the **absolute** uncertainties: $y = a \pm b \Rightarrow \Delta y = \Delta a + \Delta b$.
- **Multiply / divide** → add the **percentage** uncertainties.
- **Power** $y = a^n \rightarrow$ **multiply** the percentage by $|n|$.

Worked example. $R = V/I$ with $V = (6.0 \pm 0.1) \text{ V}$, $I = (2.0 \pm 0.1) \text{ A}$: $R = 3.0 \Omega$; $\%V = 1.7\%$, $\%I = 5.0\%$, so $\%R = 6.7\%$, i.e. $R = (3.0 \pm 0.2) \Omega$.

■ Significant figures in a calculated answer

Give a calculated result to the **same number of significant figures as the least precise measurement** it came from — no more. $\rho = \frac{m}{V}$ from a mass of 31.3 g (3 s.f.) and a side of 1.53 cm (3 s.f.) is quoted to **3 s.f.**; writing 8.7194×10^3 claims a precision the measurements do not have. When a question says **justify**, say which measurement set the limit.

■ Do two results agree?

An answer with an uncertainty is a **range**, not a point. Two results **agree** when their ranges **overlap**:

$$\rho_A = 8.7 \times 10^3 \text{ kg m}^{-3} \pm 4\% \Rightarrow 8.35 \times 10^3 \text{ to } 9.05 \times 10^3$$

$$\rho_B = 9.2 \times 10^3 \text{ kg m}^{-3} \pm 6\% \Rightarrow 8.65 \times 10^3 \text{ to } 9.75 \times 10^3$$

The two ranges share the band $8.65\text{--}9.05 \times 10^3$, so the samples **could** be the same material. Write both ranges down — that comparison is where the marks are, not the verdict on its own.

2 Practice

Now apply the methods above.

2.1 State the difference between a **systematic** error and a **random** error. [2]

2.2 A balance reads 0.3 g with **nothing** on it. Name this error and state how to correct a reading. [2]

2.3 A length is (2.50 ± 0.05) m. Find its **percentage uncertainty**. [1]

2.4 State the difference between **precision** and **accuracy**. [2]

2.5 A student measures a length with a metre rule as 84.2 cm. **Estimate** the absolute uncertainty in this reading, and explain your choice. [2]

2.6 A speed is calculated as 3.472 m s^{-1} from a distance of 12.5 m and a time of 3.6 s. State how many significant figures the answer should be given to, and **justify** your answer. [2]

3 Exam-style questions

3.1 Repeated readings are **close together** but **far from** the true value. They are: [1]

A	B
C	D

A accurate but not precise

B precise but not accurate

C both accurate and precise

D neither

3.2 Two lengths are $a = (40 \pm 1)$ mm and $b = (25 \pm 1)$ mm. **Determine** $a - b$ with its **absolute uncertainty**. [2]

3.3 A resistance is found from $R = V/I$, with $V = (6.0 \pm 0.1) \text{ V}$ and $I = (2.0 \pm 0.1) \text{ A}$. Find R and its **absolute uncertainty**. [4]

3.4 A cube has side $L = (2.00 \pm 0.02) \text{ cm}$. Its volume is $V = L^3$. Find the **percentage uncertainty** in V . [2]

3.5 Two samples of metal, A and B, are tested to find the density of the material each is made from.

(a) **Explain** what is meant by the **accuracy** of a measured value. [1]

(b) Sample A is a sphere. Its diameter is $d = (2.40 \pm 0.02) \text{ cm}$ and its mass is $m = (68.5 \pm 0.5) \text{ g}$. The volume of a sphere of diameter d is $V = \frac{\pi d^3}{6}$.

Show that the density of the material of sample A is about $9.5 \times 10^3 \text{ kg m}^{-3}$. [3]

(c) **Justify** the number of significant figures you have given for the density in (b). [1]

(d) **Calculate** the percentage uncertainty in the density of the material of sample A. [3]

(e) **Determine** the absolute uncertainty in the density of sample A, and write the density as $\rho \pm \Delta\rho$. [2]

(f) The density of the material of sample B is measured as $9.0 \times 10^3 \text{ kg m}^{-3} \pm 3\%$. **State and explain** whether A and B could be made from the same material. [2]

3.6 A student times **20** complete swings of a pendulum with a stopwatch and records 31.4 s. Her reaction time makes each timing uncertain by about 0.2 s.

(a) **Estimate** the percentage uncertainty in her value for the **period** of one swing. [2]

(b) Explain why timing 20 swings gives a smaller percentage uncertainty than timing one. [2]