Solutions

Each answer shows the steps, then the **result**.

2.1

- a **systematic** error shifts every reading the **same way**; repeating does not remove it
- a **random** error scatters readings **both ways**; the **mean** of repeats reduces it

2.2

- the balance reads a mass with nothing on it, so this is a **zero error** (systematic)
- **subtract** 0.3 g from every reading

2.3

- percentage uncertainty = $(\Delta x/|x|) \times 100\%$

$$\frac{0.05 \text{ m}}{2.50 \text{ m}} \times 100\% = \mathbf{2}\%$$

2.4

- **precision**: how close repeats are to **each other**
- **accuracy**: how close a reading (or the mean) is to the **true value**

2.5

- a metre rule is marked in millimetres, so one end is ± 0.05 cm
- a length uses **two** ends, so $\Delta L = \pm 0.1$ cm is also accepted if that is explained

2.6

- 3.6 s is given to **2 s.f.**; 12.5 m is 3 s.f.
- a calculated answer cannot be more precise than the least precise input, so quote **2 s.f.**: 3.5 m s^{-1}

3.1 B

- repeats that are **close together** are **precise**
- they sit **far from** the true value, so they are **not accurate**
- that pattern is a **systematic** error (e.g. a zero error)
- **A** swaps the two words

3.2

- subtract the lengths: $a - b = 40 \text{ mm} - 25 \text{ mm} = 15 \text{ mm}$
- for a difference, add the **absolute** uncertainties: $\Delta = 1 \text{ mm} + 1 \text{ mm} = 2 \text{ mm}$
- $(\mathbf{15} \pm \mathbf{2})$ mm

3.3

- $R = V/I$

$$R = \frac{6.0 \text{ V}}{2.0 \text{ A}} = 3.0 \ \Omega$$

- for a quotient, add the **percentage** uncertainties

$$\%V = \frac{0.1 \text{ V}}{6.0 \text{ V}} \times 100\% = 1.7\%, \qquad \%I = \frac{0.1 \text{ A}}{2.0 \text{ A}} \times 100\% = 5.0\%$$

- $\%R = 1.7\% + 5.0\% = 6.7\%$

$$\Delta R = 0.067 \times 3.0 \ \Omega = 0.2 \ \Omega \Rightarrow R = (\mathbf{3.0} \pm \mathbf{0.2}) \ \Omega$$

3.4

$$\%L = \frac{0.02 \text{ cm}}{2.00 \text{ cm}} \times 100\% = 1\%$$

- $V = L^3$, so multiply the percentage by 3: $\%V = 3 \times 1\% = \mathbf{3}\%$

3.5

(a) **accuracy** is how close the measured value is to the **true value**

(b) convert **both** quantities to SI first: $d = 2.40 \text{ cm} = 2.40 \times 10^{-2}$ m, $m = 68.5 \text{ g} = 68.5 \times 10^{-3}$ kg

$$V = \frac{\pi d^3}{6} = \frac{\pi (2.40 \times 10^{-2} \text{ m})^3}{6} = 7.24 \times 10^{-6} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{68.5 \times 10^{-3} \text{ kg}}{7.24 \times 10^{-6} \text{ m}^3} = 9.46 \times 10^3 \text{ kg m}^{-3} \approx \mathbf{9.5 \times 10^3} \text{ kg m}^{\mathbf{-3}}$$

(c) both d and m are given to **3 s.f.**, so ρ cannot be quoted more precisely than **3 s.f.**

(d)

$$\%m = \frac{0.5 \text{ g}}{68.5 \text{ g}} \times 100\% = 0.73\%$$

$$\%d = \frac{0.02 \text{ cm}}{2.40 \text{ cm}} \times 100\% = 0.83\%$$

- $V \propto d^3$, so $\%V = 3 \times 0.83\% = 2.5\%$
- $\%\rho = \%m + \%V = 0.73\% + 2.5\% = \mathbf{3.2}\%$

(e)

$$\Delta\rho = 0.032 \times 9.46 \times 10^3 \text{ kg m}^{-3} = 0.30 \times 10^3 \text{ kg m}^{-3}$$

- $\rho = (\mathbf{9.5} \pm \mathbf{0.3}) \times \mathbf{10^3}$ kg m$^{\mathbf{-3}}$

(f) write both as ranges

- A: $9.46 \times 10^3 \pm 3.2\% \Rightarrow 9.16 \times 10^3$ to 9.76×10^3 kg m^{-3}
- B: $9.0 \times 10^3 \pm 3\% \Rightarrow 8.73 \times 10^3$ to 9.27×10^3 kg m^{-3}
- the ranges **overlap**, so A and B **could** be the same material

3.6

(a) the 0.2 s is the uncertainty in the **total** time, not in one swing

$$\%T = \frac{0.2 \text{ s}}{31.4 \text{ s}} \times 100\% = \mathbf{0.6}\%$$

(b) the absolute uncertainty stays ≈ 0.2 s (reaction time) however many swings you time

- 20 swings make the **measured time 20 times larger**, so the same Δt is a much smaller **fraction** of it