A-LEVEL PHYSICS • EXERCISE SHEET

7.1.1 Wave graphs, phase, the oscilloscope and intensity

Name: ______________ Class: __________ Date: ____________ **Total: 37 marks**

KEY WORDS wave 波 • frequency 频率 • wavelength 波长 • progressive wave 行波 • amplitude 振幅

Objective

Build the skills to answer exam questions on **progressive waves** 行波 that sheet 7.1 only starts: reading a wave from displacement–distance and displacement–time graphs, **phase difference** 相位差 from a distance or a time, which way a particle moves, the **cathode-ray oscilloscope** 示波器, the wave equation with awkward units, and **intensity** 强度 from a point source. This sheet covers syllabus 7.1, learning outcomes 1 to 7.

You must be able to:

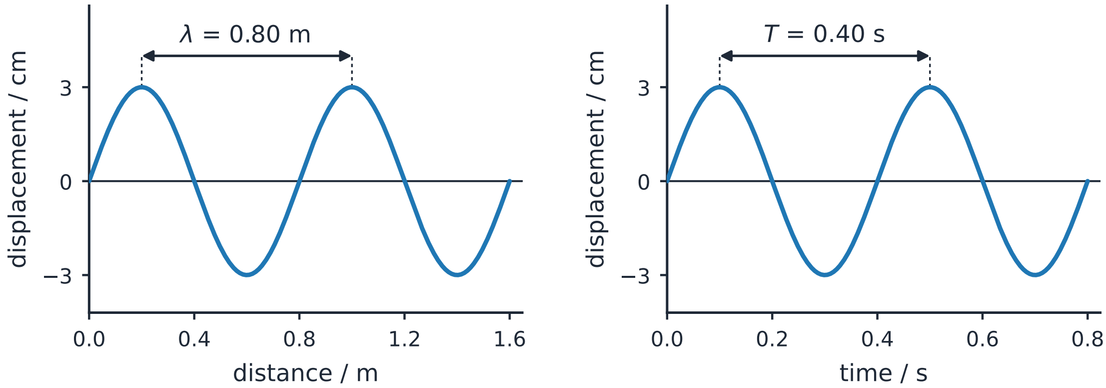
- describe what is meant by wave motion as illustrated by vibration in ropes, springs and ripple tanks
- understand and use the terms displacement, **amplitude** 振幅, phase difference, **period** 周期, **frequency** 频率, **wavelength** 波长 and speed, and read them from graphs
- understand the use of the **time-base** 时基 and **y-gain** 垂直增益 of a cathode-ray oscilloscope (CRO) to determine frequency and amplitude
- derive, using the definitions of speed, frequency and wavelength, the wave equation $v = f\lambda$
- recall and use $v = f\lambda$, including frequencies in kHz, MHz and GHz
- understand that energy is transferred by a progressive wave
- recall and use intensity = power/area and intensity $\propto$ (amplitude)^2 for a progressive wave

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Reading a wave from two graphs

A **displacement–distance** graph is a snapshot of the whole wave at one instant. A **displacement–time** graph follows one particle as the wave passes.



1. **Wavelength**, from the distance graph: the distance between neighbouring crests, $\lambda = 0.80$ m.
2. **Period**, from the time graph: the time between neighbouring crests, $T = 0.40$ s, so $f = 1/T = 2.5$ Hz.
3. **Amplitude**, from either graph: the greatest displacement from the rest position, 3.0 cm, not the 6.0 cm from crest to trough.
4. Speed: $v = f\lambda = 2.5 \times 0.80 = 2.0$ m s^{-1}.

- Check the label on the horizontal axis before you read a length from the graph: distance gives λ, time gives T.

■ Phase difference

Phase difference says how far one oscillation is ahead of another, as a fraction of a cycle. One whole cycle is 360°, or 2π rad.

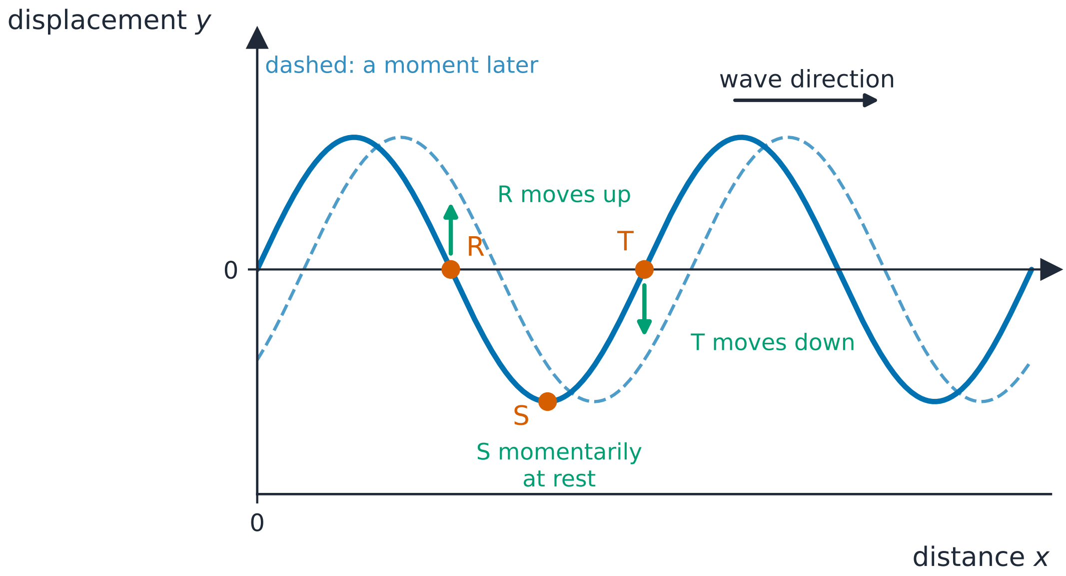
$$\Delta\phi = \frac{x}{\lambda} \times 360^\circ \qquad \text{or} \qquad \Delta\phi = \frac{t}{T} \times 360^\circ$$

For the wave above, with $\lambda = 0.80$ m and $T = 0.40$ s:

1. Two particles 0.30 m apart along the wave: $\Delta\phi = \dfrac{0.30}{0.80} \times 360^\circ = 135^\circ$, or $\frac{3}{4}\pi$ rad.
2. Two particles whose displacement–time graphs reach their crests 0.10 s apart: $\Delta\phi = \dfrac{0.10}{0.40} \times 360^\circ = 90^\circ$.
3. Particles half a wavelength apart, 0.40 m, are in **antiphase** 反相, 180°. Particles one wavelength apart, 0.80 m, are **in phase** 同相.

■ Which way does a particle move?

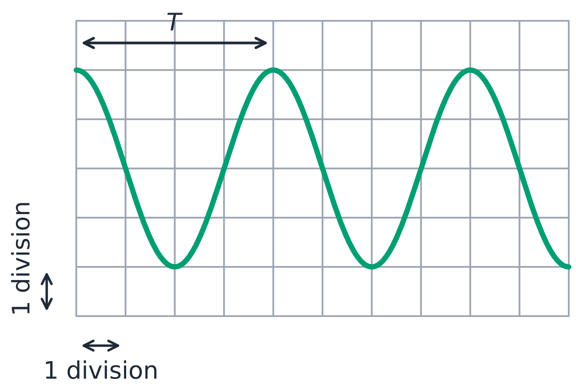
In a progressive wave each particle **oscillates** 振动 about its own rest position, and the wave travels on past it. You can see this with a pulse on a rope, compressions moving along a spring, and ripples in a **ripple tank** 水波槽, where a floating cork bobs up and down but stays in place. The wave transfers **energy** from place to place, but not matter.



1. Draw the wave again a moment later, moved a little in the direction of travel (dashed).
2. Each particle moves straight up or down to reach the dashed wave: R moves **up** and T moves **down**.
3. A particle at a **crest** 波峰 or a **trough** 波谷, such as S, is momentarily at rest.

■ Reading an oscilloscope

A cathode-ray oscilloscope shows how a voltage varies with time. The time-base is the time for each horizontal division, and the y-gain is the voltage for each vertical division.



period $= n \times$ timebase · peak voltage $= n \times$ Y-gain

The time-base is 0.20 ms per division and the y-gain is 5.0 mV per division.

1. One cycle spans 4.0 divisions, so $T = 4.0 \times 0.20$ ms $= 0.80$ ms.

2. $f = \dfrac{1}{T} = \dfrac{1}{0.80 \times 10^{-3}\ \text{s}} = 1250$ Hz.
3. From crest to trough is 4.0 divisions, so the amplitude is 2.0 divisions: $2.0 \times 5.0 =$ 10 mV.

- For a smaller uncertainty, measure the width of several cycles and divide by the number of cycles.

■ The wave equation, with awkward units

Speed = distance ÷ time. In one period T the wave moves forward one wavelength λ, so

$$v = \frac{\lambda}{T} = f\lambda \qquad \text{because } f = \frac{1}{T}$$

1. A radio station broadcasts at 97.5 MHz, and radio waves travel at 3.00×10^8 m s^{-1}: $\lambda = \dfrac{3.00 \times 10^8}{97.5 \times 10^6} = 3.08$ m.
2. Ultrasound of frequency 2.0 MHz travels at 1540 m s^{-1} in soft tissue: $\lambda = \dfrac{1540}{2.0 \times 10^6} = 7.7 \times 10^{-4}$ m $= 0.77$ mm.
3. A sound wave has a period of 5.0 ms and a wavelength of 1.7 m: $v = \dfrac{1.7}{5.0 \times 10^{-3}} =$ 340 m s^{-1}.

- Convert first: 1 kHz = 10^3 Hz, 1 MHz = 10^6 Hz, 1 GHz = 10^9 Hz and 1 ms = 10^{-3} s.

■ Intensity

The intensity of a wave is the power passing through each unit area at right angles to the direction of travel: $I = P/A$, in W m^{-2}. For a progressive wave $I \propto x_0^2$, where x_0 is the amplitude.

A small lamp is a **point source** 点源 that radiates 60 W of light evenly in all directions. The light spreads over a sphere of area $4\pi r^2$.

1. At 2.0 m: $I = \dfrac{60}{4\pi \times 2.0^2} = 1.19$ W m^{-2}.
2. At 4.0 m the same power is spread over four times the area, so $I = 1.19/4 =$ 0.30 W m^{-2}.
3. $I \propto x_0^2$, so a quarter of the intensity means half the amplitude.
4. The wave carries energy: a solar panel of area 0.50 m^2 facing sunlight of intensity 800 W m^{-2} receives $800 \times 0.50 = 400$ W, which is 4000 J in 10 s.

2 Practice

Now use the methods above.

2.1 A wave on a spring has a wavelength of 24 cm and a frequency of 12.5 Hz. Calculate its speed. [2]

2.2 A radio station broadcasts at 1.2 MHz. Radio waves travel at 3.0×10^8 m s^{-1}. Calculate the wavelength. [2]

2.3 Two points on a wave are 15 cm apart, and the wavelength is 60 cm. Calculate the phase difference between the oscillations of the two points. [2]

2.4 An oscilloscope shows exactly two cycles of a signal across 10 divisions. The time-base is set to 0.20 ms per division. Calculate the frequency of the signal. [2]

2.5 A laser beam of power 2.0 mW has a cross-sectional area of 1.5 mm^2. Calculate the intensity of the beam. [2]

2.6 A progressive wave passes along a stretched spring. State what the wave transfers along the spring, and describe the motion of one coil. [2]

3 Exam-style questions

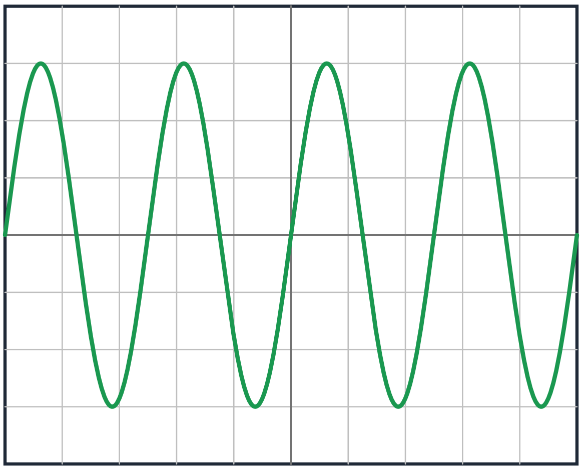
3.1 Two points on a progressive wave are 0.12 m apart. The phase difference between their oscillations is 60°. What is the wavelength? [1]

A 0.020 m
B 0.36 m
C 0.72 m
D 7.2 m

3.2 The amplitude of a progressive wave falls to one third of its value, and the frequency does not change. What fraction of the original intensity remains? [1]

A $\frac{1}{3}$
B $\frac{1}{6}$
C $\frac{1}{9}$
D $\frac{1}{27}$

3.3 A signal generator drives a loudspeaker and is also connected to a cathode-ray oscilloscope. The trace is shown, and each square is one division. The time-base is set to 0.20 ms per division and the y-gain to 10 mV per division.



(a) Determine the period of the signal. [2]

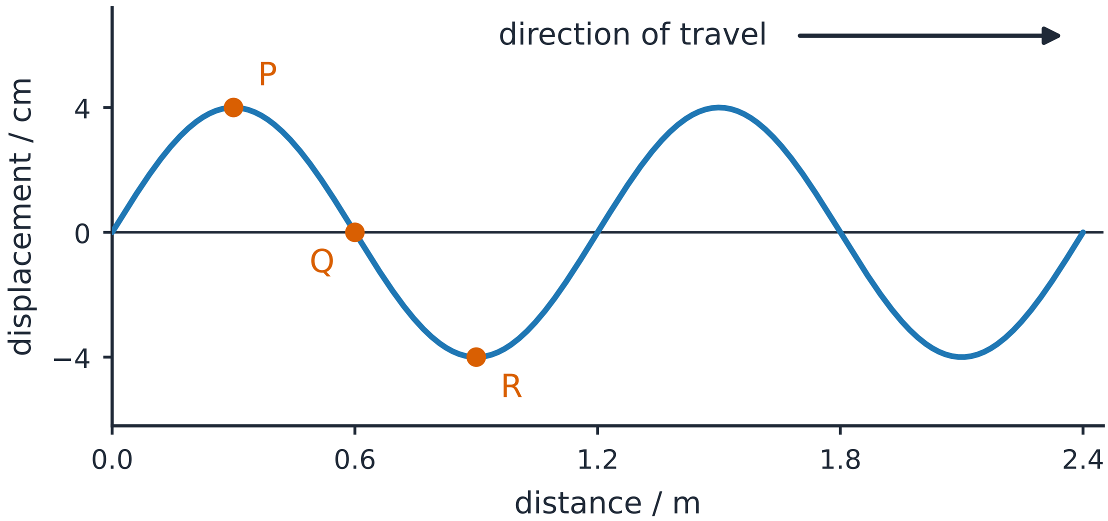
(b) Calculate the frequency of the sound from the loudspeaker. [1]

(c) Determine the amplitude of the signal. [1]

(d) The speed of sound is 340 m s^{-1}. Calculate the wavelength of the sound. [2]

(e) The output of the signal generator is turned down at the same frequency, and the amplitude of the trace falls to 2.0 divisions. The amplitude of the sound is proportional to the amplitude of the signal. Calculate $\frac{\text{new intensity}}{\text{original intensity}}$ for the sound. [2]

3.4 A transverse wave travels to the right along a rope at 3.0 m s^{-1}. The graph shows the displacement of the rope against distance at one instant. P is at a crest, Q has zero displacement and R is at a trough.



(a) Calculate the frequency of the wave. [2]

(b) State the phase difference between the oscillations of P and R. [1]

(c) State the direction in which Q is moving at this instant, and explain how you know. [2]

(d) State the displacement of P a quarter of a period later. [1]

(e) The amplitude of the wave is increased to 6.0 cm at the same frequency. Calculate $\frac{\text{new intensity}}{\text{original intensity}}$. [1]

3.5 A small loudspeaker radiates sound evenly in all directions. The sound power is 0.80 W.

(a) Define intensity. [1]

(b) Show that the intensity of the sound 5.0 m from the loudspeaker is about 2.5×10^{-3} W m^{-2}. [2]

(c) State the distance from the loudspeaker at which the intensity is one quarter of this value. [1]

(d) The amplitude of the sound wave 5.0 m from the loudspeaker is a. Determine the amplitude 10 m from the loudspeaker, in terms of a. [2]

3.6 Using the definitions of speed, frequency and wavelength, derive the wave equation $v = f\lambda$. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $v = f\lambda$

$$v = 12.5 \times 0.24 = \mathbf{3.0}\ \mathrm{m\ s^{-1}}$$

2.2

- $\lambda = v/f$

$$\lambda = \frac{3.0 \times 10^8}{1.2 \times 10^6} = \mathbf{250}\ \mathrm{m}$$

2.3

- $\Delta\phi = \dfrac{x}{\lambda} \times 360°$
- $\Delta\phi = \dfrac{15}{60} \times 360° = \mathbf{90}°$

2.4

- one cycle spans $10/2 = 5.0$ divisions, so $T = 5.0 \times 0.20$ ms $= 1.0$ ms
- $f = 1/T = \mathbf{1.0 \times 10^3}$ Hz

2.5

- $I = P/A$ with $A = 1.5 \times 10^{-6}\ \mathrm{m^2}$

$$I = \frac{2.0 \times 10^{-3}}{1.5 \times 10^{-6}} = \mathbf{1.3 \times 10^3}\ \mathrm{W\ m^{-2}}$$

2.6

- the wave transfers energy along the spring
- each coil oscillates about its own rest position, and does not travel along with the wave

3.1 C

- $\lambda = \dfrac{360°}{\Delta\phi} \times x = 6 \times 0.12 = 0.72$ m
- **A** turns the ratio upside down

3.2 C

- intensity $\propto$ (amplitude)^2, so the intensity falls to $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
- **A** treats the intensity as proportional to the amplitude

3.3

(a) one cycle spans 2.5 divisions

- $T = 2.5 \times 0.20$ ms $= \mathbf{0.50}$ ms

(b) $f = 1/T = 1/(0.50 \times 10^{-3}) = \mathbf{2.0 \times 10^3}$ Hz

(c) amplitude = 3.0 divisions × 10 mV = **30** mV

(d) $\lambda = v/f$

$$\lambda = \frac{340}{2000} = \mathbf{0.17} \text{ m}$$

(e) intensity $\propto$ (amplitude)^2

$$\frac{I_{\text{new}}}{I_{\text{original}}} = \left(\frac{2.0}{3.0}\right)^2 = \mathbf{0.44}$$

3.4

(a) from the graph $\lambda = 1.2$ m

$$f = \frac{v}{\lambda} = \frac{3.0}{1.2} = \mathbf{2.5} \text{ Hz}$$

(b) **180°**, or π rad: P and R are half a wavelength apart

(c) **upwards**

- the wave moves to the right, so the crest now at P moves towards Q; drawing the wave a moment later shows the rope at Q above the axis

(d) **zero**: in a quarter of a period P moves from the crest back to its rest position

(e) intensity $\propto$ (amplitude)^2: $\left(\frac{6.0}{4.0}\right)^2 = \mathbf{2.25}$

3.5

(a) intensity is the power per unit area, where the area is at right angles to the direction of travel of the wave

(b) the power spreads over a sphere of area $4\pi r^2$

$$I = \frac{0.80}{4\pi \times 5.0^2} = \mathbf{2.5 \times 10^{-3}} \text{ W m}^{-2}$$

(c) **10** m: twice the distance spreads the power over four times the area

(d) intensity $\propto$ (amplitude)^2, so one quarter of the intensity means half the amplitude

- amplitude = **a/2**

3.6

- speed = distance ÷ time, and in one period T the wave moves one wavelength λ, so $v = \lambda/T$
- frequency is the number of oscillations per unit time, so $f = 1/T$ and $v = f\lambda$