A-LEVEL PHYSICS • EXERCISE SHEET

6.1 Stress and strain

Name: ____________ Class: ________ Date: __________ **Total: 27 marks**

KEY WORDS young modulus 杨氏模量 • spring constant 劲度系数 • stress 应力 • extension 伸长量 • strain 应变

Objective

Build the skills to answer exam questions on **stress and strain** — Hooke's law, the spring constant, and the Young modulus.

You must be able to:

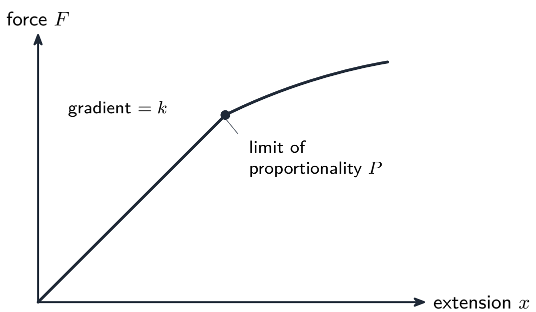
- use **Hooke's law** $F = kx$ and find k from a graph
- define and use **stress**, **strain** and the **Young modulus**
- describe the experiment to measure the **Young modulus** of a wire

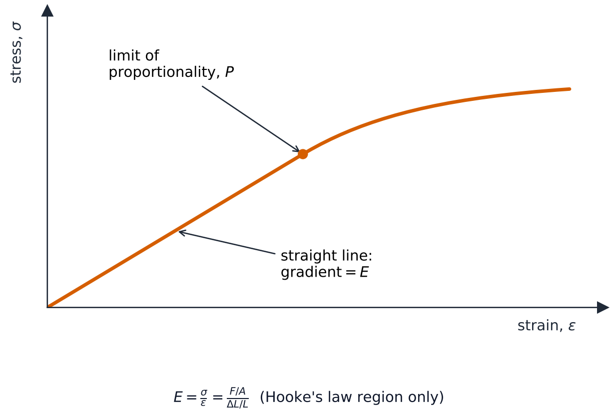
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Hooke's law and spring constant

For small loads, **extension is proportional to load**: $F = kx$, so the **spring constant** $k = F/x$ (unit N m^{-1}). On a force–extension graph the **gradient is** k in the straight region, up to the **limit of proportionality**:





The stress-strain graph for a material

Worked example. A spring extends 4.0 cm under a 2.0 N load: $k = F/x = 2.0/0.040 = 50$ N m^{-1}.

Stress, strain, Young modulus

$$\text{stress } \sigma = \frac{F}{A} \text{ (Pa)}, \qquad \text{strain } \varepsilon = \frac{x}{L} \text{ (no unit)}, \qquad E = \frac{\sigma}{\varepsilon} \text{ (Pa)}.$$

Young modulus experiment (wire): measure the original length L and diameter ($\rightarrow$ area A); add known loads; measure the extension x (e.g. with a marker and ruler/travelling microscope); plot stress against strain; E is the gradient of the straight part.

2 Practice

Now apply the methods above.

2.1 State **Hooke's law**. [1]

2.2 A spring extends 4.0 cm when a 2.0 N load is hung from it. Find the **spring constant**. [2]

2.3 Define **stress** and **strain**. [2]

2.4 State the unit of the **Young modulus**. [1]

3 Exam-style questions

3.1 In the straight region, the **gradient** of a force–extension graph gives the: [1]

- **A** Young modulus
- **B** spring constant
- **C** stress
- **D** strain

3.2 A wire of length 2.0 m and cross-sectional area 1.0×10^{-6} m^2 stretches by 1.0 mm under a 100 N load. Calculate (a) the stress, (b) the strain, (c) the Young modulus. [4]

3.3 Describe an experiment to measure the **Young modulus** of a metal wire. [4]

3.4 Explain what is meant by the **limit of proportionality**. [1]

3.5 A wire has length L, cross-sectional area A, Young modulus E and resistivity ρ.

(a) **Define** the **Young modulus** of a material. [1]

(b) **State** an expression, in terms of some or all of L, A, E and ρ, for the resistance R_0 of the wire. [1]

(c) **Show that** the spring constant k_0 of the wire is given by $k_0 = \dfrac{EA}{L}$. [2]

(d) The wire is now stretched by a steadily increasing force F, staying within its limit of proportionality. In the space below, **sketch** two graphs: the variation with F of the **resistance** R of the wire, and the variation with F of its **spring constant** k. **Label** each graph and its axes. [3]

(e) A copper wire of diameter 1.6 mm has a resistance of 0.034 Ω. The resistivity of copper is 1.7×10^{-8} Ω m. **Show that** the length of the wire is about 3.8 m. [2]

(f) The Young modulus of copper is 1.3×10^{11} Pa. **Determine** the spring constant of this wire. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **extension is proportional to the load** (up to the limit of proportionality)

2.2

- $k = F/x$

$$k = \frac{2.0 \text{ N}}{0.040 \text{ m}} = \mathbf{50} \text{ N m}^{\mathbf{-1}}$$

2.3

- **stress** = force per unit cross-sectional area (F/A)
- **strain** = extension per unit original length (x/L)

2.4

- **pascal**, Pa (N m^{-2})

3.1 B

- in the straight region $F = kx$, so the gradient is $F/x = k$, the **spring constant**
- **A** is the trap: the Young modulus is the gradient of a **stress–strain** graph

3.2

(a)

$$\sigma = \frac{F}{A} = \frac{100 \text{ N}}{1.0 \times 10^{-6} \text{ m}^2} = \mathbf{1.0 \times 10^{8}} \text{ Pa}$$

(b)

$$\varepsilon = \frac{x}{L} = \frac{1.0 \times 10^{-3} \text{ m}}{2.0 \text{ m}} = \mathbf{5.0 \times 10^{-4}}$$

(c)

$$E = \frac{\sigma}{\varepsilon} = \frac{1.0 \times 10^{8} \text{ Pa}}{5.0 \times 10^{-4}} = \mathbf{2.0 \times 10^{11}} \text{ Pa}$$

3.3 any four of

- measure original length L and diameter ($\rightarrow$ area A)
- hang the wire and add known loads
- measure the extension with a marker against a fixed scale
- plot stress against strain; E is the gradient of the straight part

3.4

- the point beyond which **extension is no longer proportional to load** (the F–x line stops being straight)

3.5

(a) the **ratio of stress to strain** (below the limit of proportionality)

(b) $R_0 = \rho L/A$

(c) $k = F/x$ and $E = (F/A)/(x/L) = FL/(Ax)$

- so $E = k_0 L/A \Rightarrow k_0 = EA/L$

(d)

- **resistance**: straight line of **positive** gradient from $(0, R_0)$ — stretching makes the wire longer and thinner
- **spring constant**: **horizontal** line from $(0, k_0)$ — k is constant below the limit of proportionality
- both axes labelled, each line labelled

(e) diameter 1.6 mm $\Rightarrow$ radius 0.80 mm

$$A = \pi r^2 = \pi(0.80 \times 10^{-3}\ \text{m})^2 = 2.01 \times 10^{-6}\ \text{m}^2$$

$$L = \frac{R_0 A}{\rho} = \frac{(0.034\ \Omega)(2.01 \times 10^{-6}\ \text{m}^2)}{1.7 \times 10^{-8}\ \Omega\,\text{m}} \approx \mathbf{3.8}\ \text{m}$$

(f) use $L = 3.8$ m from (e)

$$k_0 = \frac{EA}{L} = \frac{(1.3 \times 10^{11}\ \text{Pa})(2.01 \times 10^{-6}\ \text{m}^2)}{3.8\ \text{m}} = \mathbf{6.9 \times 10^4}\ \text{N m}^{-1}$$