6.1.1 Deformation, the force-extension graph and measuring the Young modulus

Name: ______________ Class: __________ Date: ____________ **Total: 31 marks**

KEY WORDS young modulus 杨氏模量 • extension 伸长 • spring constant 弹簧常数 • strain 应变 • stress 应力

Objective

Sheet 6.1 gave you Hooke's law and one Young-modulus substitution. This sheet is the rest of 6.1: the difference between **tensile** 拉伸 and **compressive** 压缩 **deformation** 形变, every named point on a force-extension graph and what each one means, springs joined in series and in parallel, why a **stress-strain** graph describes the **material** while force-extension describes the **specimen** 试样, and the Young modulus experiment done properly – including which measurement limits the accuracy. This sheet covers syllabus 6.1.

You must be able to:

- understand that deformation is caused by tensile or compressive forces
- use the terms **load** 负载, **extension** 伸长, compression and **limit of proportionality** 比例极限
- recall and use **Hooke's law** and the **spring constant** 弹簧常数 $k = F/x$
- define and use stress, strain and the **Young modulus** 杨氏模量
- describe an experiment to determine the Young modulus of a metal in the form of a wire

1 Worked examples

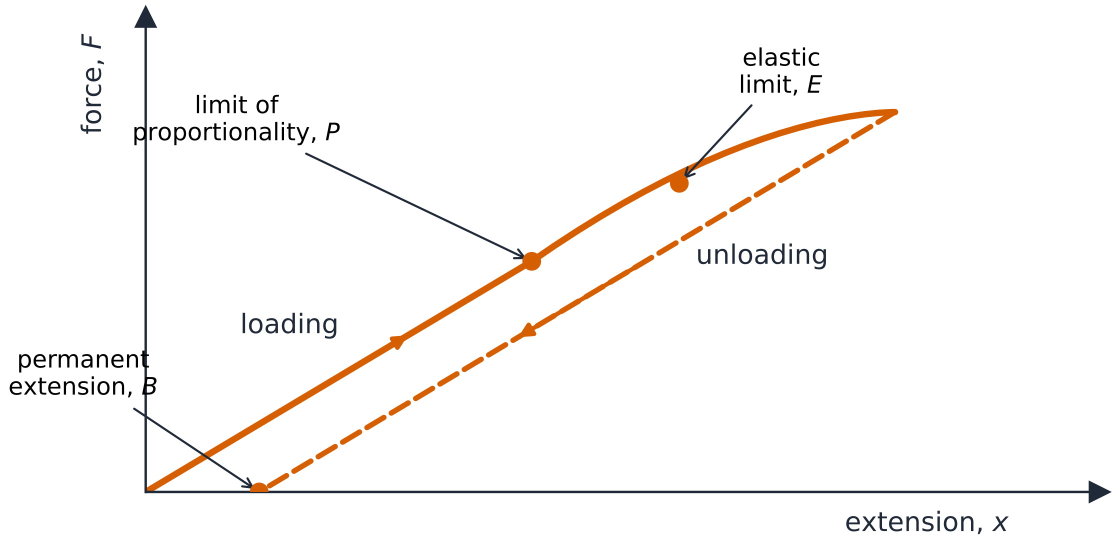
Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Two kinds of deformation, and the words for them

- A **tensile** force **stretches** an object: the forces at the two ends pull **outwards**, and the object gets **longer**. The increase in length is the **extension**.
- A **compressive** force **squashes** it: the forces push **inwards**, and the object gets **shorter**. The decrease is the **compression**.
- Either way, the forces come in a **pair**. A single force acting on a free object accelerates it; it takes two opposing forces to deform it.
- The **load** is the force applied, usually the weight of the masses hung on the spec-

imen. Do not confuse it with the mass: $W = mg$.

Every named point on a force-extension graph



Hooke up to limit of proportionality · plastic beyond yield

point	what it means
the **straight** region	extension is **proportional to** load; the specimen **obeys Hooke's law**
limit of proportionality	the point beyond which the graph is **no longer straight**; $F \propto x$ fails here
elastic limit	the point beyond which the specimen does **not** return to its original length when the load is removed; it is just past the limit of proportionality
plastic region	the specimen is **permanently deformed**; removing the load leaves an extension behind
breaking point	where it snaps

- The two limits are **different points** and the exam distinguishes them. Proportionality is about the **shape of the graph**; elasticity is about **what happens when you let go**.
- In the straight region the **gradient is the spring constant** k, so $k = \dfrac{F}{x}$. A **stiffer** specimen gives a **steeper** line.

Springs joined together

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \qquad\qquad k = k_1 + k_2$$

Do not memorise formulae for these – work them out from the two questions that matter:

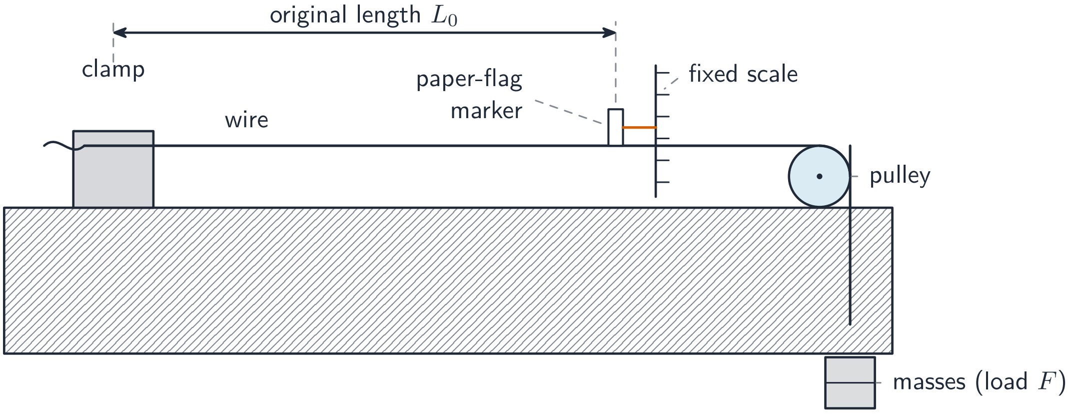
joined	what force does each spring carry?	what extension does the pair have?
in series (end to end)	the **whole** load F, each of them	each extends F/k, and the extensions **add**: total $= 2F/k$, so the pair is **less stiff**
in parallel (side by side)	the load is **shared**, so $F/2$ each	each extends $F/2k$, and that **is** the extension of the pair: **stiffer**

Stress, strain, and why we bother

$$\text{stress } \sigma = \frac{F}{A} \quad \text{(Pa)} \qquad \text{strain } \varepsilon = \frac{x}{L} \quad \text{(no unit)} \qquad E = \frac{\sigma}{\varepsilon} \quad \text{(Pa)}$$

- A **force-extension** graph describes **this specimen**: a thicker or shorter wire of the same metal gives a different line.
- Dividing the force by the **area** and the extension by the **original length** removes the specimen's dimensions, so a **stress-strain** graph describes the **material**. Every copper wire gives the same one.
- In the straight region the gradient of a stress-strain graph is the **Young modulus**, and it is a property of the material alone.
- Strain has **no unit**, so the Young modulus has the same unit as stress: **pascals**. Typical metals are around 10^{11} Pa; an answer of 10^{9} usually means a unit was missed somewhere.

■ Measuring the Young modulus of a wire



$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

so plotting **load against extension** gives a straight line of gradient $\dfrac{EA}{L}$, and

$$E = \text{gradient} \times \frac{L}{A}$$

What is measured, and with what:

quantity	**instrument**	**note**
original length L	metre rule	measured from the clamp to the marker; make it **long** so the extension is measurable
diameter d	**micrometer**, at several points and in different directions	$A = \pi d^2/4$ – the wire may not be perfectly round
extension x	marker against a **millimetre scale**, or a travelling microscope	take a reading **before** loading, as the zero
load F	known masses, $F = mg$	add **and then remove** them, checking the wire returns to its original length

- △ **The diameter carries the largest percentage uncertainty**, because it is the smallest quantity measured – and it is **squared** in the area, so its percentage uncertainty is **doubled** in A and therefore in E. Measuring it carefully matters more than anything else in the experiment.
- Use a **long, thin** wire: a larger L and a smaller A both make the extension bigger and easier to measure.

2 Practice

Now use the methods above.

2.1 State what is meant by a **tensile** force and by a **compressive** force. [2]

2.2 Explain what is meant by the **limit of proportionality**, and state how it differs from the **elastic limit**. [3]

2.3 Two identical springs, each of spring constant 25 N m^{-1}, are joined end to end and a load of 3.0 N is hung from the pair. Calculate the total extension. [3]

2.4 State what the gradient of a **stress-strain** graph represents in its straight region. [1]

2.5 Explain why a stress-strain graph describes the **material** while a force-extension graph describes only that **specimen**. [2]

2.6 A wire of diameter 0.40 mm carries a load of 25 N. Calculate the stress in the wire. [3]

3 Exam-style questions

3.1 A wire obeys Hooke's law. Which quantity is given by the gradient of a graph of stress against strain? [1]

A	B
C	D

A the spring constant of the wire
B the Young modulus of the material
C the stiffness of that particular wire
D the breaking stress of the material

3.2 A spring obeys Hooke's law up to a load of 8.0 N, at which its extension is 0.16 m.

(a) Calculate the spring constant of the spring. [2]

(b) Two such springs are connected **in parallel** and the same 8.0 N load is hung from the pair. Calculate the extension. [2]

(c) A single spring is loaded beyond its elastic limit and the load is then removed. Describe what has happened to the spring. [2]

3.3 A student determined the Young modulus of a metal using a wire of original length 2.50 m and diameter 0.38 mm. A graph of load against extension was a straight line of gradient 9.0×10^3 N m^{-1} up to the limit of proportionality.

(a) Calculate the cross-sectional area of the wire. [2]

(b) Show that the Young modulus is given by $E = \text{gradient} \times \dfrac{L}{A}$. [2]

(c) Calculate the Young modulus of the metal. [2]

(d) State which measured quantity contributes the largest percentage uncertainty to the value of E, and explain why. [2]

(e) Suggest **two** changes that would improve the accuracy of the experiment. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- a **tensile** force is a pair of forces pulling **outwards** on the ends of an object, stretching it so that it becomes longer
- a **compressive** force is a pair pushing **inwards**, squashing it so that it becomes shorter

2.2

- the **limit of proportionality** is the point beyond which the extension is **no longer proportional** to the load, so the graph stops being a straight line
- the **elastic limit** is the point beyond which the specimen no longer returns to its **original length** when the load is removed
- they are different points: the elastic limit lies just beyond the limit of proportionality

2.3

- springs in series each carry the **whole** load

$$x_{\text{each}} = \frac{F}{k} = \frac{3.0\ \text{N}}{25\ \text{N m}^{-1}} = 0.12\ \text{m}$$

- the extensions add

$$x_{\text{total}} = 2 \times 0.12\ \text{m} = \mathbf{0.24}\ \text{m}$$

2.4

- the **Young modulus** of the material

2.5

- a force-extension graph changes if the wire is made **thicker or shorter**, even for the same metal, so it describes that specimen
- dividing force by **area** and extension by **original length** removes the specimen's dimensions, so every specimen of the same material gives the **same** stress-strain graph

2.6

- the radius is half the diameter, so

$$A = \pi r^2 = \pi(0.20 \times 10^{-3}\ \text{m})^2 = 1.26 \times 10^{-7}\ \text{m}^2$$

$$\sigma = \frac{F}{A} = \frac{25\ \text{N}}{1.26 \times 10^{-7}\ \text{m}^2} = \mathbf{2.0 \times 10^8}\ \text{Pa}$$

3.1 B

- the gradient of stress against strain is $\dfrac{\sigma}{\varepsilon}$, which is the **Young modulus**
- **A** and **C** are the gradient of a **force-extension** graph and depend on the wire's dimensions; **D** is a single point on the graph, not its gradient

3.2

(a)

- use $k = F/x$

$$k = \frac{8.0\ \text{N}}{0.16\ \text{m}} = \mathbf{50}\ \text{N m}^{-1}$$

(b)

- in parallel the load is **shared**, so each spring carries 4.0 N

$$x = \frac{4.0\ \text{N}}{50\ \text{N m}^{-1}} = \mathbf{0.080}\ \text{m}$$

- the extension of the pair is the extension of each one, not the sum

(c)

- the spring has been **permanently deformed**
- it does not return to its original length: some extension remains after the load is removed

3.3

(a)

- the radius is half the diameter

$$A = \pi \left(\frac{0.38 \times 10^{-3}\ \text{m}}{2}\right)^2 = \mathbf{1.13} \times \mathbf{10^{-7}}\ \text{m}^2$$

(b)

- start from the definitions

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L} = \frac{F}{x} \times \frac{L}{A}$$

- and $\frac{F}{x}$ is the **gradient** of the load-extension graph, so $E = \text{gradient} \times \frac{L}{A}$

(c)

- substitute

$$E = 9.0 \times 10^3\ \text{N m}^{-1} \times \frac{2.50\ \text{m}}{1.13 \times 10^{-7}\ \text{m}^2} = \mathbf{2.0} \times \mathbf{10^{11}}\ \text{Pa}$$

(d)

- the **diameter** of the wire
- it is the smallest quantity measured, so its percentage uncertainty is the largest, and it is **squared** in the area, which **doubles** that percentage uncertainty in A and therefore in E

(e)

- any **two** of: measure the diameter at **several points** along the wire and in different directions, and take a mean; use a **longer** wire so the extension is larger; use a **travelling microscope** or a vernier scale to read the extension; check the wire returns to its original length as the loads are removed; take readings for both increasing and decreasing loads