A-LEVEL PHYSICS • EXERCISE SHEET

5.2 Gravitational potential energy and kinetic energy

Name: ____________ Class: ________ Date: __________ **Total: 34 marks**

KEY WORDS energy 能量 • kinetic energy 动能 • gravitational potential energy 重力势能 • mass 质量 • work 功

Objective

Build the skills to answer exam questions on **gravitational potential energy and kinetic energy** — the formulae, their derivations, and energy exchange.

You must be able to:

- use $\Delta E_{\mathrm{P}} = mg\Delta h$ and derive it from $W = Fs$
- use $E_{\mathrm{K}} = \frac{1}{2}mv^2$ and derive it from the equations of motion
- solve problems where GPE and KE exchange

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ GPE change

Lifting mass m slowly through height Δh needs an upward force mg, so the work done (= GPE gained) is

$$\Delta E_{\mathrm{P}} = W = (mg)(\Delta h) = mg\Delta h.$$

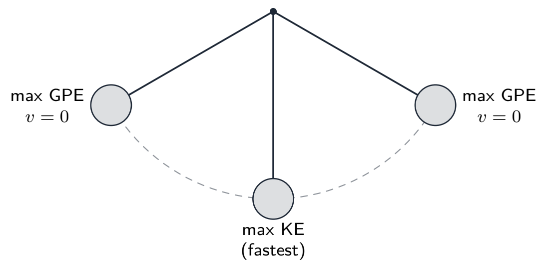
■ Kinetic energy

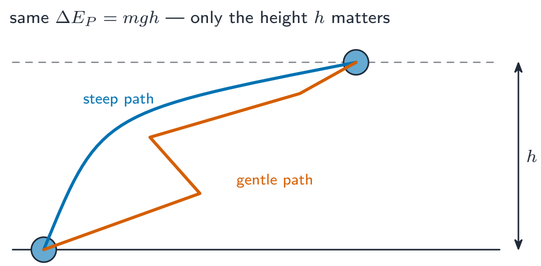
A force F accelerates a mass from rest to speed v over distance s. With $u = 0$, $v^2 = 2as$, so $s = v^2/2a$, and

$$E_{\mathrm{K}} = W = Fs = (ma)\frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

■ Energy exchange (pendulum)

A pendulum bob trades GPE and KE: **max GPE** (and $v = 0$) at the ends, **max KE** at the bottom:





GPE depends only on height, not the path

Energy is conserved: $mgh = \frac{1}{2}mv^2$, so the bob's speed at the bottom is $v = \sqrt{2gh}$.

■ Energy accounting in a chain of transfers

The hardest energy questions are not one formula: they follow the energy through **several** stores and ask you to keep a running total. A falling block that lands on a spring is the standard case:

$$E_{\text{elastic, max}} = E_{\mathrm{K}\text{ on contact}} + \Delta E_{\mathrm{P}\text{ during compression}}$$

The second term is easy to forget — the block keeps falling *while* it compresses the spring, so it loses **more** potential energy after contact.

Two more results the exam builds on top:

- the **area under a force-compression graph** is the elastic potential energy, so for a spring obeying Hooke's law $E = \frac{1}{2}F_0x_0$, which gives $F_0 = \dfrac{2E}{x_0}$;
- at maximum compression the spring pushes up with F_0 while the weight still pulls down, so the **resultant** force is $F_0 - mg$ and the acceleration is $\dfrac{F_0 - mg}{m}$ — large, upward, and the moment the block starts back up.

2 Practice

Now apply the methods above.

2.1 Write the formula for **GPE change** and for **kinetic energy**. [2]

2.2 Find the **kinetic energy** of a 1200 kg car moving at 15 m s^{-1}. [2]

2.3 Find the **GPE gained** by a 2.0 kg book lifted 1.5 m ($g = 9.81$ m s^{-2}). [2]

2.4 At the **bottom** of a pendulum swing, which energy is at a maximum? [1]

3 Exam-style questions

3.1 An object's speed **doubles**. Its kinetic energy: [1]

- **A** doubles
- **B** halves
- **C** becomes four times larger
- **D** stays the same

3.2 Derive $E_K = \frac{1}{2}mv^2$ using the equations of motion. [3]

3.3 A 0.50 kg ball is dropped from a height of 1.8 m. Calculate its speed just before it hits the ground (ignore air resistance). [3]

3.4 A pendulum bob is released from a point 0.20 m above its lowest point. Calculate its **maximum speed**. [2]

3.5 A 0.20 kg ball is dropped from a height of 1.6 m and rebounds to a height of 1.2 m. Calculate the energy **transferred to other forms** during the bounce. [3]

3.6 A vertical spring stands on a horizontal surface with its lower end fixed. Its mass is negligible. A block of mass 4.0 kg falls onto the spring and is brought to rest as the spring compresses. The block has kinetic energy 72 J at the instant it touches the spring.

(a) **Calculate** the speed of the block as it touches the spring. [2]

(b) While the spring compresses, the gravitational potential energy of the block falls by a further 12 J. **Show that** the maximum compression x_0 of the spring is about 0.31 m. [2]

(c) All the energy lost by the block becomes elastic potential energy in the spring. **Determine** the maximum elastic potential energy stored. [1]

(d) The spring obeys Hooke's law, so the force it exerts is proportional to the compression. **Show that** the maximum force F_0 the spring exerts on the block is about 540 N. [2]

(e) At maximum compression, **determine**

(i) the resultant force on the block, [2]

(ii) the acceleration of the block. [2]

(f) **Sketch** a graph to show how the **kinetic energy** of the block varies with the compression of the spring, from the instant of contact to maximum compression. **Label** both axes. [2]

(g) The same block is dropped from a greater height onto the same spring. **Explain** why the maximum compression is greater, and why the maximum acceleration is greater too. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\Delta E_P = mg\Delta h$
- $E_K = \frac{1}{2}mv^2$

2.2

- $E_K = \frac{1}{2}mv^2$

$$E_K = \tfrac{1}{2}(1200\text{ kg})(15\text{ m s}^{-1})^2 = \mathbf{1.35 \times 10^5}\text{ J}$$

2.3

- $\Delta E_P = mg\Delta h$

$$\Delta E_P = (2.0\text{ kg})(9.81\text{ m s}^{-2})(1.5\text{ m}) = \mathbf{29}\text{ J}$$

2.4

- **kinetic energy**

3.1 C

- $E_K \propto v^2$, so doubling v makes KE **four** times larger

3.2

- work done $W = Fs = mas$
- from rest, $v^2 = 2as \Rightarrow s = v^2/(2a)$

$$W = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2 = E_K$$

3.3

- $\frac{1}{2}mv^2 = mgh$

$$v = \sqrt{2gh} = \sqrt{2(9.81\text{ m s}^{-2})(1.8\text{ m})} = \mathbf{5.9}\text{ m s}^{-1}$$

3.4

- from rest, $v^2 = 2gh$

$$v = \sqrt{2(9.81\text{ m s}^{-2})(0.20\text{ m})} = \mathbf{2.0}\text{ m s}^{-1}$$

3.5

- GPE before: $mgh_1 = (0.20\text{ kg})(9.81\text{ m s}^{-2})(1.6\text{ m}) = 3.1$ J
- GPE after: $mgh_2 = (0.20\text{ kg})(9.81\text{ m s}^{-2})(1.2\text{ m}) = 2.4$ J
- energy transferred $= 3.1\text{ J} - 2.4\text{ J} = \mathbf{0.79}$ J

3.6

(a) $E_K = \frac{1}{2}mv^2$

$$72\text{ J} = \tfrac{1}{2}(4.0\text{ kg})v^2 \Rightarrow v = \sqrt{36\text{ m}^2\text{ s}^{-2}} = \mathbf{6.0}\text{ m s}^{-1}$$

(b) the block falls a further x_0 while compressing the spring, so $\Delta E_P = mgx_0$

$$x_0 = \frac{12\text{ J}}{(4.0\text{ kg})(9.81\text{ m s}^{-2})} = \mathbf{0.31}\text{ m}$$

(c) all of it goes into the spring: $72\text{ J} + 12\text{ J} = \mathbf{84}$ J

(d) energy stored = area under the F–x graph = a triangle: $E = \frac{1}{2}F_0x_0$

$$F_0 = \frac{2 \times 84\text{ J}}{0.306\text{ m}} = \mathbf{5.5 \times 10^2}\text{ N}$$

(e)(i) spring **up** with F_0, weight **down**: $W = (4.0\text{ kg})(9.81\text{ m s}^{-2}) = 39$ N

- resultant $= 549\text{ N} - 39\text{ N} = \mathbf{5.1 \times 10^2}$ N **upwards**

(e)(ii)

$$a = \frac{F}{m} = \frac{510\text{ N}}{4.0\text{ kg}} = \mathbf{1.3 \times 10^2}\text{ m s}^{-2}$$

upwards

(f)

- axes labelled kinetic energy / J against compression / m
- starts at 72 J when $x = 0$, falls to zero at $x_0 = 0.31$ m, getting steeper as the spring force grows

(g) a greater drop height $\Rightarrow$ **more KE** at contact $\Rightarrow$ more energy stored; $E = \frac{1}{2}kx^2$ so x_0 is larger

- $F_0 \propto x_0$, so $a = (F_0 - mg)/m$ is larger too