A-LEVEL PHYSICS • EXERCISE SHEET

# 5.2.1 Potential energy, kinetic energy and energy transfers

Name: ______________ Class: __________ Date: ____________ **Total: 35 marks**

**KEY WORDS** kinetic energy 动能 • gravitational potential energy 重力势能 • energy 能量 • thermal energy 热能 • momentum 动量

## Objective

Build the skills to answer exam questions on **gravitational potential energy 重力势能** and **kinetic energy 动能** that sheet 5.2 only starts: both derivations, why only the height matters, the change in kinetic energy when a force acts, comparing kinetic energies, finding a speed from energy, and energy transfers when air resistance acts. This sheet covers syllabus 5.2, learning outcomes 1 to 4.

**You must be able to:**

- derive, using $W = Fs$, the formula $\Delta E_{\mathrm{P}} = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field
- recall and use the formula $\Delta E_{\mathrm{P}} = mg\Delta h$ for gravitational potential energy changes in a uniform gravitational field, whatever path the object takes
- derive, using the equations of motion, the formula for kinetic energy $E_{\mathrm{K}} = \frac{1}{2}mv^2$, and the change in kinetic energy when a force acts
- recall and use $E_{\mathrm{K}} = \frac{1}{2}mv^2$, including ratios of kinetic energies and energy changes with and without air resistance

## 1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

### ■ Deriving the change in gravitational potential energy

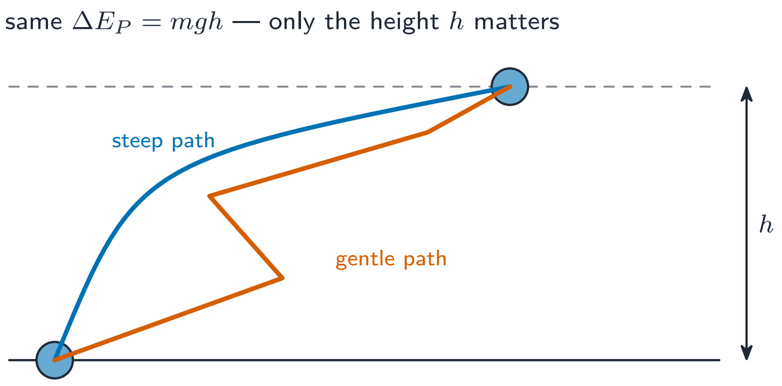
To lift an object of mass $m$ at a steady speed, an upward force equal to its weight, $F = mg$, must act on it. As the object rises through a height $\Delta h$, this force does work

$$W = Fs = mg\Delta h$$

and this work is stored as gravitational potential energy: $\Delta E_{\mathrm{P}} = mg\Delta h$.

- The derivation needs a **uniform** gravitational field, where $g$ is the same at every height. This is true near the Earth's surface, but not for a rocket that rises hundreds of kilometres.

- Only the vertical height matters, not the path taken.




A bag of mass 12 kg is raised through 1.5 m.

1. Lifted straight up: $\Delta E_\mathrm{P} = 12 \times 9.81 \times 1.5 = 177$ J.
2. Pushed without friction up a ramp 4.0 m long that rises 1.5 m: the force needed is the component of the weight along the ramp, $12 \times 9.81 \times \dfrac{1.5}{4.0} = 44.1$ N.
3. The work done on the ramp is $44.1 \times 4.0 = 177$ J: a smaller force over a longer distance gives the same energy.

## ■ Deriving the kinetic energy

A resultant force $F$ acts on a mass $m$ over a displacement $s$, and its speed increases from $u$ to $v$. Using $F = ma$ and the equation of motion $v^2 = u^2 + 2as$:

$$Fs = ma \times \frac{v^2 - u^2}{2a} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

- Starting from rest ($u = 0$), all the work done becomes kinetic energy: $E_\mathrm{K} = \frac{1}{2}mv^2$.
- The work done by the resultant force equals the **change** in kinetic energy.

A car of mass 900 kg speeds up from 12 m s$^{-1}$ to 20 m s$^{-1}$ over a distance of 80 m.

1. $\Delta E_\mathrm{K} = \frac{1}{2} \times 900 \times (20^2 - 12^2) = 1.152 \times 10^5$ J.
2. The resultant force is $F = \dfrac{\Delta E_\mathrm{K}}{s} = \dfrac{1.152 \times 10^5}{80} = 1440$ N.
3. Check with the equations of motion: $a = \dfrac{20^2 - 12^2}{2 \times 80} = 1.6$ m s$^{-2}$, and $F = 900 \times 1.6 = 1440$ N.

- Take care: $\frac{1}{2}m(v^2 - u^2)$ is **not** the same as $\frac{1}{2}m(v - u)^2$.

## Comparing kinetic energies

- $E_K \propto v^2$: tripling the speed multiplies the kinetic energy by 9.
- At the same speed, $E_K \propto m$.
- Using momentum $p = mv$ (topic 3): $E_K = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$.

A car of mass 1000 kg moves at 20 m s$^{-1}$. A truck of mass 4000 kg has the same momentum.

1. The car has $p = 1000 \times 20 = 2.0 \times 10^4$ kg m s$^{-1}$ and $E_K = \frac{1}{2} \times 1000 \times 20^2 = 2.0 \times 10^5$ J.
2. The truck moves at $v = 2.0 \times 10^4/4000 = 5.0$ m s$^{-1}$, so $E_K = \frac{1}{2} \times 4000 \times 5.0^2 = 5.0 \times 10^4$ J.
3. The same momentum with four times the mass gives one quarter of the kinetic energy, as $E_K = p^2/2m$ predicts.

## Finding a speed from energy

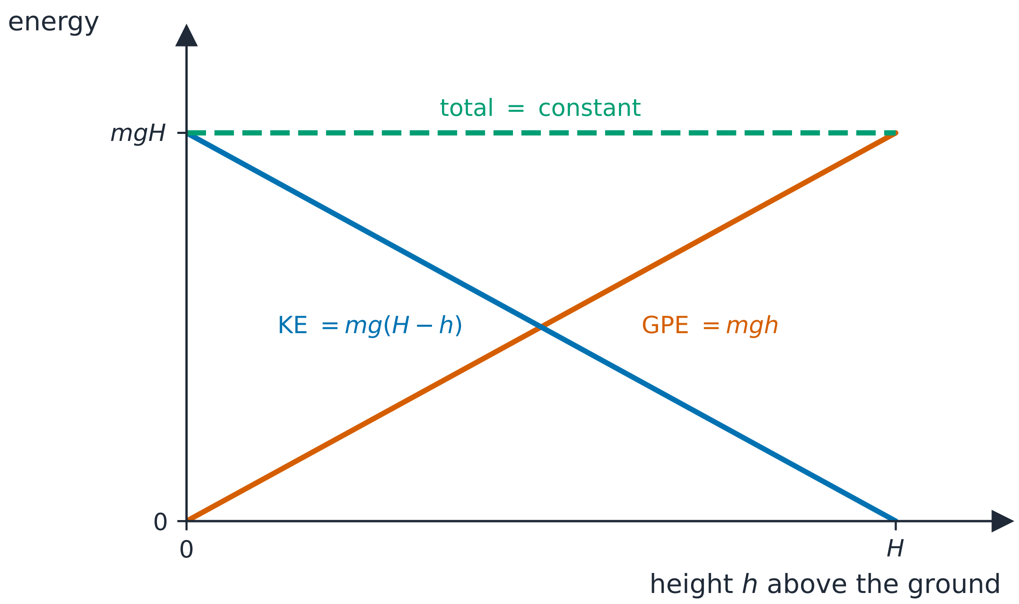
A stone is thrown horizontally at 8.0 m s$^{-1}$ from the top of a cliff 20 m high. Air resistance is negligible.

1. Energy is conserved: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg\Delta h$. The mass cancels.
2. $v = \sqrt{u^2 + 2g\Delta h} = \sqrt{8.0^2 + 2 \times 9.81 \times 20} = 21.4$ m s$^{-1}$.

- Energy gives the **speed** at the bottom whatever the direction of the throw. For the direction of the velocity, or the time taken, use the equations of motion instead.

## Energy against height, and air resistance

When a ball falls with no air resistance, it loses gravitational potential energy and gains the same amount of kinetic energy, so the total stays constant:




A ball of mass 0.20 kg is dropped from a height of 5.0 m and reaches the ground with 8.0 J of kinetic energy.

1. Its gravitational potential energy at the start is $0.20 \times 9.81 \times 5.0 = 9.81$ J.
2. With no air resistance it would land with 9.81 J of kinetic energy, so $9.81 - 8.0 = 1.8$ J was transferred to thermal energy.
3. The average air resistance was $\dfrac{1.81}{5.0} = 0.36$ N.

- With air resistance the total energy of the ball is not constant: the lower the ball, the less total energy it has left.

## 2 Practice

Now use the methods above.

**2.1** A cable car of total mass 1200 kg climbs through a vertical height of 18 m. Calculate the gain in its gravitational potential energy. [2]

**2.2** A tennis ball of mass 0.058 kg is served at 45 m s$^{-1}$. Calculate its kinetic energy. [2]

**2.3** A car of mass 1100 kg speeds up from 20 m $s^{-1}$ to 30 m $s^{-1}$.

(a) State the factor by which its kinetic energy increases. [1]

(b) Calculate the increase in its kinetic energy. [2]

**2.4** Water moves very slowly over the top of a waterfall and falls through 32 m. Ignoring air resistance, calculate its speed at the bottom. [2]

**2.5** State the condition for the formula $\Delta E_P = mg\Delta h$ to be used. [1]

## 3 Exam-style questions

**3.1** Two objects X and Y have the same momentum. The mass of X is twice the mass of Y. What is the ratio $\dfrac{\text{kinetic energy of X}}{\text{kinetic energy of Y}}$? [1]

**A** $\frac{1}{4}$
**B** $\frac{1}{2}$
**C** 2
**D** 4

**3.2** A ball dropped from rest from a height $h$ reaches a speed $v$. Air resistance is negligible. From what height must the ball be dropped to reach a speed $2v$? [1]

**A** $\sqrt{2}\,h$
**B** $2h$
**C** $4h$
**D** $8h$

**3.3** A footballer kicks a ball of mass 0.45 kg from rest. The ball leaves the foot at 26 m $s^{-1}$.

(a) Use the equations of motion to show that the kinetic energy of an object of mass $m$, accelerated from rest to a speed $v$, is $\frac{1}{2}mv^2$. [3]

(b) The foot pushes on the ball over a distance of 0.20 m. Calculate the average force on the ball. [2]

(c) At the highest point of its path the ball moves horizontally at 15 m $s^{-1}$. Ignoring air resistance, calculate the height of this point above the point where the ball was kicked. [3]

**3.4** A pole vaulter of mass 70 kg sprints along the runway at 9.0 m $s^{-1}$, then uses the pole to change her kinetic energy into gravitational potential energy.

(a) Calculate her kinetic energy just before she plants the pole. [2]

(b) Assuming that all of this energy becomes gravitational potential energy, calculate how far her centre of gravity could rise. [2]

(c) Her centre of gravity starts 1.0 m above the ground, and she only just clears a bar at 4.8 m. Suggest two reasons why she does not reach the height predicted by (b). [2]

(d) After clearing the bar, she falls from rest at 4.8 m onto a landing mat whose top surface is 0.80 m above the ground. Treating her as a point mass, calculate her speed as she reaches the mat. [2]

**3.5** A hiker of mass 60 kg walks 3.2 km along a mountain path, from a height of 1200 m to a height of 1650 m.

(a) Starting from the work done by a force, derive the formula for the change in gravitational potential energy near the Earth's surface. [2]

(b) Explain why this formula cannot be used for a rocket that rises 2000 km above the Earth. [1]

(c) Calculate the gain in gravitational potential energy of the hiker. [2]

(d) The hiker's muscles transfer food energy into useful work with an efficiency of 25%. Calculate the food energy used to gain this potential energy. [2]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- $\Delta E_\text{P} = mg\Delta h$

$$\Delta E_\text{P} = 1200 \times 9.81 \times 18 = \mathbf{2.12 \times 10^5}\ \text{J}$$

**2.2**

- $E_\text{K} = \frac{1}{2}mv^2$

$$E_\text{K} = \tfrac{1}{2} \times 0.058 \times 45^2 = \mathbf{59}\ \text{J}$$

**2.3**

(a) $E_\text{K} \propto v^2$, so the factor is $(30/20)^2 = \mathbf{2.25}$

(b) $\Delta E_\text{K} = \frac{1}{2} \times 1100 \times (30^2 - 20^2)$

- $\Delta E_\text{K} = \mathbf{2.75 \times 10^5}\ \text{J}$

**2.4**

- $\frac{1}{2}mv^2 = mg\Delta h$

$$v = \sqrt{2 \times 9.81 \times 32} = \mathbf{25}\ \text{m s}^{-1}$$

**2.5**

- the gravitational field must be uniform: $g$ does not change with height, as near the Earth's surface for small changes in height

**3.1 B**

- $E_\text{K} = p^2/2m$, so with the same momentum $E_\text{K} \propto 1/m$ and X, with twice the mass, has half the kinetic energy
- **C** would be true for the same speed, not the same momentum

**3.2 C**

- $\frac{1}{2}mv^2 = mgh$, so $h \propto v^2$ and doubling $v$ needs four times the height
- **B** assumes that $h$ is proportional to $v$

**3.3**

(a) the work done by the force is $W = Fs$ with $F = ma$

- from rest $v^2 = 2as$, so $s = v^2/2a$
- $W = ma \times \dfrac{v^2}{2a} = \frac{1}{2}mv^2$, and this work becomes the kinetic energy

(b) $E_\text{K} = \frac{1}{2} \times 0.45 \times 26^2 = 152$ J

- $F = \dfrac{E_\text{K}}{s} = \dfrac{152}{0.20} = \mathbf{760}$ N

(c) kinetic energy at the top $= \frac{1}{2} \times 0.45 \times 15^2 = 50.6$ J, so the gain in potential energy is $152.1 - 50.6 = 101.5$ J

- $mg\Delta h = 101.5$

$$\Delta h = \frac{101.5}{0.45 \times 9.81} = \mathbf{23} \text{ m}$$

**3.4**

(a) $E_K = \frac{1}{2} \times 70 \times 9.0^2$

- $E_K = \mathbf{2.8 \times 10^3}$ J

(b) $mg\Delta h = 2835$ J

$$\Delta h = \frac{2835}{70 \times 9.81} = \mathbf{4.1} \text{ m}$$

(c) she still has kinetic energy at the top, because she must keep moving horizontally over the bar

- some energy is transferred to thermal energy and sound in the pole and in her body, and by air resistance

(d) $\frac{1}{2}mv^2 = mg\Delta h$ with $\Delta h = 4.8 - 0.80 = 4.0$ m

$$v = \sqrt{2 \times 9.81 \times 4.0} = \mathbf{8.9} \text{ m s}^{\mathbf{-1}}$$

**3.5**

(a) to raise the mass at a steady speed, a force equal to its weight $mg$ acts through the vertical distance $\Delta h$

- the work done $W = Fs = mg\Delta h$ is stored as gravitational potential energy, so $\Delta E_P = mg\Delta h$

(b) $g$ decreases as the height increases, so the field is not uniform over 2000 km

(c) $\Delta h = 1650 - 1200 = 450$ m

$$\Delta E_P = 60 \times 9.81 \times 450 = \mathbf{2.6 \times 10^5} \text{ J}$$

(d) food energy = useful work ÷ efficiency = $2.65 \times 10^5/0.25$

- food energy = $\mathbf{1.1 \times 10^6}$ J