

5.1 Energy conservation

Name: _____ Class: _____ Date: _____

Total: 26 marks

KEY WORDS power 功率 • work 功 • efficiency 效率 • energy 能量
• conservation of energy 能量守恒

Objective

Build the skills to answer exam questions on **energy conservation** — work, the conservation principle, efficiency and power.

You must be able to:

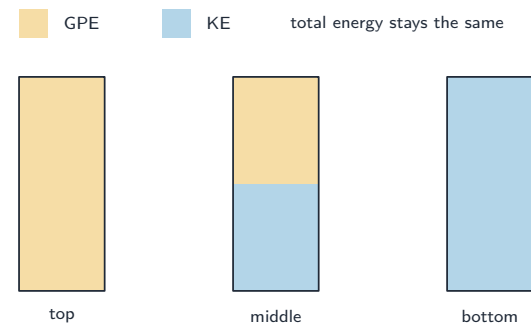
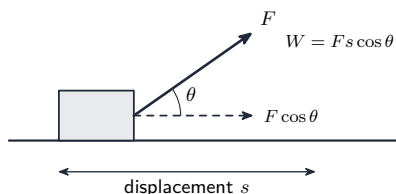
- use **work done** $= F s \cos \theta$
- apply **conservation of energy** (e.g. GPE $\rightarrow$ KE)
- use **efficiency** and **power** ($P = W/t$ and $P = Fv$)

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Work done

Work is done when a force moves along its own line: $W = F s \cos \theta$. Only the component **along** the displacement does work:



Energy is conserved as it transfers

- $\theta = 0^\circ$: $W = F s$ (max); $\theta = 90^\circ$: $W = 0$ (e.g. the normal force on a flat road); $\theta = 180^\circ$: $W = -F s$ (e.g. friction).

■ Conservation of energy

Energy is never made or destroyed. On a **smooth** ramp, GPE $\rightarrow$ KE:

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}.$$

■ Efficiency and power

$$\text{efficiency} = \frac{\text{useful energy out}}{\text{total energy in}}, \quad P = \frac{W}{t}, \quad P = Fv.$$

$$P = Fv \text{ comes from } P = W/t = Fs/t = F(s/t) = Fv.$$

■ Efficiency, and where the wasted energy goes

Efficiency compares what you **want** with what you **paid for**, as energies or as powers:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}$$

It has **no unit**, and it can be written as a decimal or a percentage — but never above

1. Two habits win the marks:

- **Work out the useful output first**, from what the machine actually achieves (mgh for a lift, Fs for a drag, $\frac{1}{2}mv^2$ for a launch).
- **The wasted energy is input minus useful output**, and the exam wants to know *where it went*: thermal energy in the motor windings and bearings, sound, and in a lift the kinetic energy left over if it does not stop.

Worked example. A pump raises 600 kg of water through 12 m in 30 s, drawing 3.0 kW.

$$P_{\text{useful}} = \frac{mgh}{t} = \frac{600 \times 9.81 \times 12}{30} = 2354 \text{ W}, \quad \text{efficiency} = \frac{2354}{3000} = 0.78$$

2 Practice

Now apply the methods above.

2.1 Define the **work done** by a force. [1]

2.2 A child pulls a sledge 5.0 m with a 20 N force at 60° to the ground. Find the **work done**. [2]

2.3 Define **power** and give its unit. [1]

2.4 State the **principle of conservation of energy**. [1]

3 Exam-style questions

3.1 A force acts at 90° to the direction of motion. The work it does is: [1]

- A maximum
- B Fs
- C zero
- D negative

3.2 A ball is released from rest at the top of a **smooth** ramp 1.2 m high. Calculate its speed at the bottom. ($g = 9.81 \text{ m s}^{-2}$.) [3]

3.3 A motor lifts a 50 kg load through 8.0 m in 10 s.

(a) Calculate the **useful power** output. [2]

(b) If the input power is 500 W, calculate the **efficiency**. [2]

3.4 Derive $P = Fv$ from $P = W/t$. [2]

3.5 A crane is used on a building site.

(a) **Define** the **work done** by a force. [1]

(b) A block of mass m is raised vertically at constant speed through a height Δh . **Derive** an expression, in terms of m and Δh , for the change in gravitational potential energy ΔE_P of the block. State the meaning of any other symbol you use. [2]

(c) The crane raises a steel beam of mass 450 kg at constant speed through a vertical height of 24 m, taking 40 s.

(i) **Calculate** the work done on the beam. [2]

(ii) **Calculate** the useful output power of the crane. [2]

(iii) The input power to the crane is 3.6 kW. **Calculate** its efficiency. [2]

(iv) **Determine** the energy wasted during the 40 s, and **state** one form it takes. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- work = force $\times$ displacement **in the direction of the force** ($W = Fs \cos \theta$)

2.2

- $W = Fs \cos \theta$

$$W = (20 \text{ N})(5.0 \text{ m})(\cos 60^\circ) = 20 \times 5.0 \times 0.50 = \mathbf{50 \text{ J}}$$

2.3

- power is the **work done (energy transferred) per unit time**, unit **watt** ($W = \text{J s}^{-1}$)

2.4

- energy cannot be **created or destroyed**, only **transferred**; the total energy of a closed system is constant

3.1 C

- $W = Fs \cos \theta$ and $\cos 90^\circ = 0$, so a force at right angles to the motion does **no work**
- B** is only true when $\theta = 0$

3.2

- energy conservation: $mgh = \frac{1}{2}mv^2$ (the m cancels)

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m s}^{-2})(1.2 \text{ m})} = \mathbf{4.9 \text{ m s}^{-1}}$$

3.3

(a) useful work = mgh

$$W = (50 \text{ kg})(9.81 \text{ m s}^{-2})(8.0 \text{ m}) = 3924 \text{ J}$$

$$P = \frac{W}{t} = \frac{3924 \text{ J}}{10 \text{ s}} = \mathbf{392 \text{ W}}$$

(b)

$$\text{efficiency} = \frac{392 \text{ W}}{500 \text{ W}} = 0.78 = \mathbf{78\%}$$

3.4

- $P = W/t = Fs/t$
- $s/t = v$, so $P = Fv$

3.5

- (a) the product of the force and the **displacement in the direction of the force**
- (b) constant speed $\Rightarrow$ upward force = weight mg

- gain in PE = work done by that force, so $\Delta E_p = mg\Delta h$

(c)(i)

$$W = mg\Delta h = (450 \text{ kg})(9.81 \text{ m s}^{-2})(24 \text{ m}) = \mathbf{1.1 \times 10^5 \text{ J}}$$

(c)(ii)

$$P = \frac{W}{t} = \frac{1.06 \times 10^5 \text{ J}}{40 \text{ s}} = \mathbf{2.6 \times 10^3 \text{ W}}$$

(c)(iii)

$$\text{efficiency} = \frac{P_{\text{useful}}}{P_{\text{input}}} = \frac{2650 \text{ W}}{3600 \text{ W}} = 0.74 = \mathbf{74\%}$$

(c)(iv) input energy = $(3600 \text{ W})(40 \text{ s}) = 1.44 \times 10^5 \text{ J}$

- wasted = $1.44 \times 10^5 \text{ J} - 1.06 \times 10^5 \text{ J} = \mathbf{3.8 \times 10^4 \text{ J}}$
- it becomes **thermal energy** in the motor, gearbox and cable