5.1.1 Work, efficiency and power

Name: ______________ Class: __________ Date: ____________ **Total: 35 marks**

KEY WORDS power 功率 • work 功 • energy 能量 • efficiency 效率 • gravitational potential energy 重力势能

Objective

Build the skills to answer exam questions on **work 功**, **efficiency 效率** and **power 功率** that sheet 5.1 only starts: the work done by each force on an object, the energy that resistive forces take away, machines made of several stages, the power to lift a load up a slope, and $P = Fv$ on level ground and on slopes. This sheet covers syllabus 5.1, learning outcomes 1 to 7.

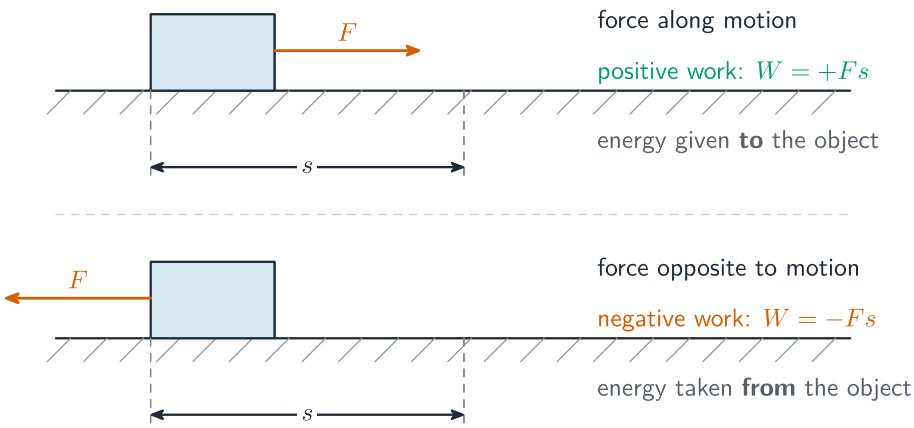
You must be able to:

- understand the concept of work, and recall and use work done = force × displacement in the direction of the force, including forces that do negative work or no work
- recall and apply the principle of **conservation of energy 能量守恒** when resistive forces act
- recall and understand that the efficiency of a system is the ratio of useful energy output from the system to the total energy input
- use the concept of efficiency to solve problems, including a machine made of several stages
- define power as work done per unit time
- solve problems using $P = W/t$, including the power to lift a load up a slope
- derive $P = Fv$ and use it to solve problems, on level ground and on slopes

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

> ### ■ Work done by each force
>
> The **work done** by a force is the force multiplied by the **displacement 位移** in the direction of the force: $W = Fs\cos\theta$, where θ is the angle between the force and the displacement. Work is measured in joules.



A suitcase is pulled 15 m along a level floor by a strap at 30° above the horizontal. The pull in the strap is 60 N and the friction force is 40 N.

1. The pull does $W = 60 \times 15 \times \cos 30° = +779$ J. Only its horizontal component, $60 \cos 30° = 52$ N, does work.
2. Friction acts against the motion, so $\theta = 180°$ and $W = 40 \times 15 \times \cos 180° = -600$ J.
3. The weight and the normal contact force act at 90° to the displacement, so they do **no work**.
4. Of the 779 J given by the pull, 600 J becomes **thermal energy** 热能 at the floor, and the other 179 J increases the **kinetic energy** 动能 of the case.

■ Energy transfers with a resistive force

The principle of conservation of energy: energy cannot be created or destroyed, only transferred from one form to another, so the total energy stays the same.

A skier of mass 70 kg starts from rest and travels 120 m down a slope, dropping 25 m vertically. At the bottom her speed is 16 m s^{-1}.

1. Loss of **gravitational potential energy** 重力势能: $mg\Delta h = 70 \times 9.81 \times 25 = 17\,168$ J.
2. Gain in kinetic energy: $\frac{1}{2}mv^2 = \frac{1}{2} \times 70 \times 16^2 = 8960$ J.
3. Energy is conserved, so the difference was transferred by the resistive forces (friction and air resistance) to thermal energy: $17\,168 - 8960 = 8208$ J.
4. This is the work done against the average resistive force F over the 120 m path:

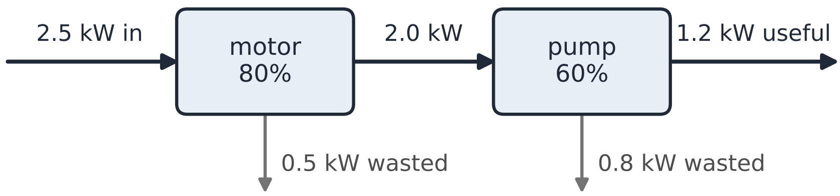
$$F = \frac{8208 \text{ J}}{120 \text{ m}} = 68 \text{ N}$$

- Use the distance **along the path** for the resistive force, but the **vertical** drop for the potential energy.

Efficiency in stages

The **efficiency** of a system is the ratio of the useful energy output to the total energy input. The same ratio holds for powers. Efficiency has no unit and is never more than 1.

An electric motor with an efficiency of 80% drives a pump with an efficiency of 60%. The motor takes 2.5 kW from the supply.

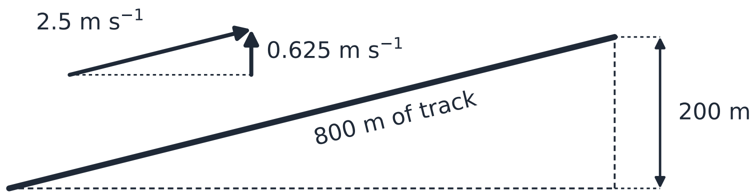


1. The motor's useful output is $0.80 \times 2.5 = 2.0$ kW. This is the **input** to the pump.
2. The pump's useful output is $0.60 \times 2.0 = 1.2$ kW.
3. The overall efficiency is $\dfrac{1.2}{2.5} = 0.48$, which is the product 0.80×0.60.
4. The total wasted power is $2.5 - 1.2 = 1.3$ kW, mostly thermal energy in the motor and the pump.

Power to lift a load up a slope

Power is the work done per unit time, $P = W/t$, measured in watts: $1\ \text{W} = 1\ \text{J s}^{-1}$.

A ski lift carries 16 skiers, each of mass 80 kg, at a steady $2.5\ \text{m s}^{-1}$ along an 800 m track that rises 200 m.



1. The skiers gain potential energy only by rising. Their vertical speed is $2.5 \times \dfrac{200}{800} = 0.625\ \text{m s}^{-1}$.
2. Each second they rise 0.625 m, so the power needed to lift them is

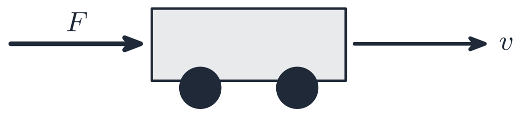
$$P = \frac{mg\Delta h}{t} = 16 \times 80 \times 9.81 \times 0.625 = 7850\ \text{W}$$

3. Friction in the machinery needs another 2.0 kW, so the useful output of the motor is $7.85 + 2.0 = 9.85$ kW.
4. The motor is 80% efficient, so its input power is $9.85/0.80 = 12.3$ kW.

■ Deriving and using P = Fv

A force F moves an object at a constant velocity v through a displacement s in a time t:

$$P = \frac{W}{t} = \frac{Fs}{t} = F \times \frac{s}{t} = Fv$$



$P = Fv$

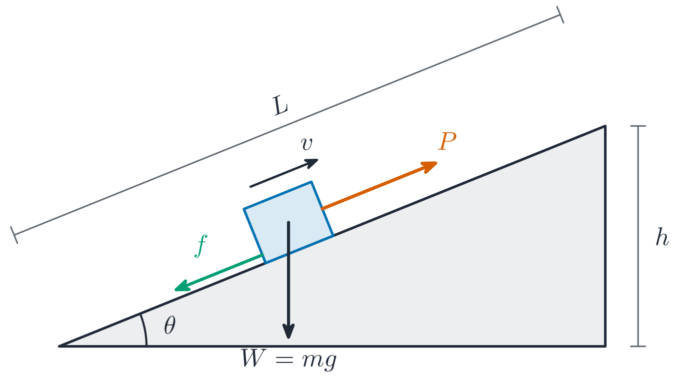
A car of mass 1200 kg has a maximum useful power of 45 kW.

1. **Top speed.** At top speed the driving force equals the resistive force, 1500 N, so $v = P/F = 45\,000/1500 = 30$ m s^{-1}.
2. **Driving force at a lower speed.** At 20 m s^{-1} on full power, $F = P/v = 45\,000/20 = 2250$ N.
3. If the resistive force at this speed is 900 N, the resultant force is $2250 - 900 = 1350$ N, so the acceleration is $a = 1350/1200 = 1.1$ m s^{-2}.

- On constant power, a larger speed means a smaller driving force: $F = P/v$.

■ Power up a slope

A winch pulls a crate of mass 80 kg up a ramp at a constant speed $v = 0.50$ m s^{-1}. The ramp is at $\theta = 15°$ to the horizontal, and the friction force on the crate is $f = 120$ N.



1. The component of the weight down the ramp is $mg\sin\theta = 80 \times 9.81 \times \sin 15° = 203$ N.
2. At constant speed the forces along the ramp balance, so the pull is $P = 203 + 120 = 323$ N.
3. Power = force × velocity, so the winch delivers $323 \times 0.50 = 162$ W.
4. Energy check: the crate gains potential energy at $203 \times 0.50 = 102$ W, and friction turns $120 \times 0.50 = 60$ W into thermal energy. $102 + 60 = 162$ W.

2 Practice

Now use the methods above.

2.1 A horse on the bank pulls a boat 12 m along a canal with a rope. The rope is at 35° to the direction in which the boat moves, and the tension in it is 250 N. Calculate the work done by the tension. [2]

2.2 A box is pushed 8.0 m at a constant speed along a level floor by a horizontal force of 45 N.

(a) Calculate the work done by the push. [1]

(b) State the work done by the friction force, and what happens to this energy. [2]

2.3 An electric motor takes 1200 W from the supply and gives a useful output power of 900 W.

(a) Calculate its efficiency. [1]

(b) Calculate the energy it wastes in 5.0 minutes. [2]

2.4 The winch on a rescue helicopter raises a person of mass 72 kg at a steady 0.80 m s^{-1}. Calculate the power the winch delivers to the person. [2]

2.5 Explain why a car moving at a constant speed along a level road still needs power from its engine. [2]

3 Exam-style questions

3.1 A constant force does 1200 J of work on a box as the box moves 40 m along a horizontal floor. The force acts at 60° to the floor. What is the size of the force? [1]

A 30 N
B 35 N
C 60 N
D 120 N

3.2 A motor with an efficiency of 60% lifts a load of mass 50 kg vertically at a constant speed of 0.30 m s^{-1}. What is the input power to the motor? [1]

A 88 W
B 147 W
C 245 W
D 490 W

3.3 A car of mass 1200 kg travels at a constant speed of 15 m s^{-1} up a straight road that makes an angle of 5.0° with the horizontal. The total resistive force on the car is 800 N.

(a) Show that the component of the weight of the car down the slope is about 1.0 kN. [2]

(b) Calculate the driving force on the car. [1]

(c) Use $P = Fv$ to calculate the useful output power of the engine. [2]

(d) The engine has an efficiency of 30%. Calculate the rate at which the fuel supplies energy to the engine. [2]

(e) State the principle of conservation of energy, and use it to explain what happens to the energy that the fuel supplies each second. [2]

3.4 A hydroelectric power station uses water that falls through a height of 120 m at a rate of 1500 kg per second. The turbines are 85% efficient and the generators are 95% efficient.

(a) Calculate the rate at which the water loses gravitational potential energy. [2]

(b) Calculate the overall efficiency of the power station. [1]

(c) Calculate the electrical output power. [2]

(d) Calculate the energy wasted in one hour. [2]

3.5 A tractor pulls a trailer across a level field at a constant speed of 4.0 m s^{-1}. The tow bar is at 20° to the horizontal and the tension in it is 3.0 kN.

(a) Derive $P = Fv$ for a constant force F that moves an object at a constant speed v in the direction of the force. [2]

(b) Calculate the rate at which the tension does work on the trailer. [2]

(c) The useful output power of the tractor's engine is 18 kW. Suggest why this is larger than your answer to (b). [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $W = Fs\cos\theta$

$$W = 250 \times 12 \times \cos 35° = \mathbf{2.46 \times 10^3}\ \text{J}$$

2.2

(a) $W = Fs = 45 \times 8.0 = \mathbf{360}$ J

(b) friction does $-\mathbf{360}$ J of work

- the speed is constant, so there is no gain in kinetic energy: the energy becomes thermal energy in the box and the floor

2.3

(a) efficiency $= 900/1200 = \mathbf{0.75}$, or 75%

(b) wasted power $= 1200 - 900 = 300$ W

- energy $= 300 \times 5.0 \times 60 = \mathbf{9.0 \times 10^4}$ J

2.4

- at a steady speed the pull of the cable equals the weight: $F = mg = 72 \times 9.81 = 706$ N

$$P = Fv = 706 \times 0.80 = \mathbf{565}\ \text{W}$$

2.5

- resistive forces such as air resistance and friction act against the motion and do negative work on the car, taking energy from it
- the driving force must do positive work at the same rate to keep the kinetic energy constant; the energy ends as thermal energy in the air and the tyres

3.1 C

- $W = Fs\cos\theta$, so $F = \dfrac{1200}{40 \times \cos 60°} = 60$ N
- **A** leaves out $\cos 60°$ and **B** uses $\sin 60°$

3.2 C

- useful power $= mgv = 50 \times 9.81 \times 0.30 = 147$ W, and the input is $147/0.60 = 245$ W
- **A** multiplies by the efficiency instead of dividing, and **B** is the useful output

3.3

(a) component $= mg\sin\theta$

$$1200 \times 9.81 \times \sin 5.0° = \mathbf{1.03 \times 10^3}\ \text{N} \approx 1.0\ \text{kN}$$

(b) driving force $= 1026 + 800 = \mathbf{1.83 \times 10^3}$ N

(c) $P = Fv$

$$P = 1826 \times 15 = \mathbf{2.7 \times 10^4} \text{ W}$$

(d) input = useful output ÷ efficiency

$$\frac{2.74 \times 10^4}{0.30} = \mathbf{9.1 \times 10^4} \text{ W}$$

(e) energy cannot be created or destroyed, only transferred from one form to another, so the total stays the same

- of the 91 kW, $1026 \times 15 = 15$ kW raises the car's gravitational potential energy, $800 \times 15 = 12$ kW becomes thermal energy through the resistive forces, and the other 64 kW is wasted as thermal energy in the engine

3.4

(a) power = mass per second $\times g \times h$

$$1500 \times 9.81 \times 120 = \mathbf{1.77 \times 10^6} \text{ W}$$

(b) $0.85 \times 0.95 = \mathbf{0.81}$

(c) electrical power $= 0.8075 \times 1.766 \times 10^6$

- $= \mathbf{1.43 \times 10^6}$ W

(d) wasted power $= 1.766 \times 10^6 - 1.426 \times 10^6 = 3.40 \times 10^5$ W

- energy $= 3.40 \times 10^5 \times 3600 = \mathbf{1.22 \times 10^9}$ J

3.5

(a) $W = Fs$, so $P = W/t = Fs/t$

- $s/t = v$, so $P = Fv$

(b) the component of the tension along the motion is $3000 \cos 20° = 2.82 \times 10^3$ N

$$P = 2819 \times 4.0 = \mathbf{1.13 \times 10^4} \text{ W}$$

(c) the engine also does work against the resistive forces on the tractor itself, such as friction at its wheels and air resistance, and some energy is wasted in its gears