A-LEVEL PHYSICS • EXERCISE SHEET

4.3 Density and pressure

Name: ____________ Class: ________ Date: __________ **Total: 31 marks**

KEY WORDS density 密度 • pressure 压强 • upthrust 浮力 • hydrostatic pressure 流体静压强 • atmospheric pressure 大气压强

Objective

Build the skills to answer exam questions on **density and pressure** — including hydrostatic pressure and upthrust.

You must be able to:

- use **density** $\rho = m/V$ and **pressure** $p = F/A$
- derive and use **hydrostatic pressure** $\Delta p = \rho g \Delta h$
- find **upthrust** with $F = \rho g V$ (Archimedes' principle)
- get a density from **measured** side lengths, and its **percentage uncertainty**
- decide whether an object **floats**, and how much of it sits below the surface

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Density and pressure

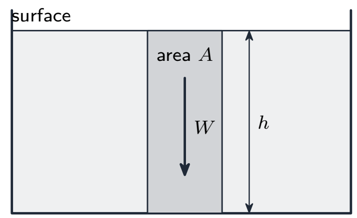
$$\rho = \frac{m}{V} \text{ (kg m}^{-3}\text{)}, \qquad p = \frac{F}{A} \text{ (Pa = N m}^{-2}\text{)}.$$

Water has $\rho = 1000$ kg m^{-3}.

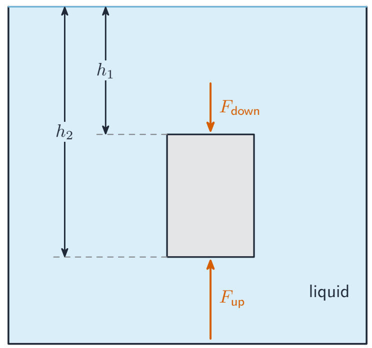
■ Hydrostatic pressure

A column of liquid (area A, depth h) has weight $W = \rho A h g$ pressing on its base, so the **extra** pressure is

$$\Delta p = \frac{W}{A} = \rho g h.$$



$\Delta p = \rho g h$



Upthrust arises from the pressure difference with depth

It depends only on **depth and density** — not on the container's shape. (For total pressure, add atmospheric pressure $\approx 1.0 \times 10^5$ Pa.)

■ Upthrust

A submerged object feels a larger pressure on its **bottom** than its top — this pressure difference gives an upward **upthrust**:

$$F = \rho g V,$$

where V is the volume of fluid displaced (Archimedes' principle).

■ Density from measurements, with its uncertainty

An exam density is rarely handed to you: it comes from a mass and three measured sides, each with an uncertainty.

Worked example. $m = (0.243 \pm 0.001)$ kg; $x = (5.41 \pm 0.01)$ cm, $y = (11.09 \pm 0.01)$ cm, $z = (1.62 \pm 0.01)$ cm.

$$V = xyz = 5.41 \times 11.09 \times 1.62 = 97.2 \text{ cm}^3 = 9.72 \times 10^{-5} \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{0.243}{9.72 \times 10^{-5}} = 2500 \text{ kg m}^{-3}$$

For the uncertainty, add the **percentage** uncertainties of everything multiplied or divided:

$$\frac{0.001}{0.243} + \frac{0.01}{5.41} + \frac{0.01}{11.09} + \frac{0.01}{1.62} = 0.41 + 0.18 + 0.09 + 0.62 = 1.3\%$$

The **smallest** side carries the largest percentage — that is the measurement worth improving, and examiners ask exactly that.

Floating

An object floats when the upthrust can balance its weight **before** it is fully under:

$$\rho_{\text{object}} V_{\text{total}}\, g = \rho_{\text{fluid}} V_{\text{submerged}}\, g \quad \Rightarrow \quad \frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

So ice (920 kg m^{-3}) floats in water with 92% of its volume below the surface. If $\rho_{\text{object}} > \rho_{\text{fluid}}$ the fraction would exceed 1, which is the algebra telling you it sinks.

2 Practice

Now apply the methods above.

2.1 Define **density** and give its SI unit. [1]

2.2 A block has mass 2400 kg and volume 0.30 m^3. Find its **density**. [2]

2.3 A force of 200 N acts on an area of 0.50 m^2. Find the **pressure**. [2]

2.4 State what causes the **upthrust** on a submerged object. [1]

3 Exam-style questions

3.1 The extra pressure at the bottom of a liquid column depends on: [1]

- **A** the cross-sectional area only
- **B** the depth and the density
- **C** the shape of the container
- **D** the total volume of liquid

3.2 Calculate the extra pressure due to the water at the bottom of a pool 2.5 m deep. ($\rho = 1000$ kg m^{-3}, $g = 9.81$ m s^{-2}.) [2]

3.3 A block of volume 2.0×10^{-3} m^3 is fully submerged in water ($\rho = 1000$ kg m^{-3}). Calculate the **upthrust** on it. [2]

3.4 Derive the hydrostatic pressure equation $\Delta p = \rho g h$ from the definitions of **pressure** and **density**. [3]

3.5 A student measures a rectangular metal block to find the density of the metal. The results are

quantity	**measurement**
mass m	(0.512 ± 0.001) kg
length x	(8.02 ± 0.01) cm
width y	(4.15 ± 0.01) cm
height z	(2.05 ± 0.01) cm

(a) **Define** density. [1]

(b) **Determine** the density of the metal. [3]

(c) **Determine** the percentage uncertainty in the density. [3]

(d) **State**, with a reason, which single measurement the student should improve first. [2]

(e) The block is lowered into water ($\rho = 1000$ kg m^{-3}) until it is fully submerged. **Show that** the upthrust on it is about 0.67 N, and **explain** why the block still sinks. [3]

(f) A block of wood of density 650 kg m^{-3} floats in the same water. **Determine** the fraction of its volume that lies **below** the surface. [2]

(g) A swimming pool is 25 m long, 10 m wide and 2.0 m deep. **Estimate** the force that the water exerts on the bottom of the pool, and state one assumption you make. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- density is **mass per unit volume**, unit kg m^{-3}

2.2

- $\rho = m/V$

$$\rho = \frac{2400\text{ kg}}{0.30\text{ m}^3} = \mathbf{8000}\text{ kg m}^{-3}$$

2.3

- $p = F/A$

$$p = \frac{200\text{ N}}{0.50\text{ m}^2} = \mathbf{400}\text{ Pa}$$

2.4

- the **difference in pressure** between the bottom and top of the object (the bottom is deeper)

3.1 B

- $\Delta p = \rho g \Delta h$ depends only on **density** and **depth**
- **A** / **D** (volume, area) do not appear: a narrow tube and a wide tank of the same depth give the same pressure

3.2

- $\Delta p = \rho g h$

$$\Delta p = (1000\text{ kg m}^{-3})(9.81\text{ m s}^{-2})(2.5\text{ m}) = \mathbf{2.5 \times 10^4}\text{ Pa}$$

3.3

- upthrust = weight of fluid displaced = $\rho g V$

$$F = (1000\text{ kg m}^{-3})(9.81\text{ m s}^{-2})(2.0 \times 10^{-3}\text{ m}^3) = \mathbf{19.6}\text{ N}$$

3.4

- mass of the column $m = \rho V = \rho A h$
- weight $W = mg = \rho A h g$ presses on area A

$$\Delta p = \frac{W}{A} = \rho g h$$

3.5

(a) density is **mass per unit volume**

(b) $V = 8.02\text{ cm} \times 4.15\text{ cm} \times 2.05\text{ cm} = 68.2\text{ cm}^3$

- convert: $V = 6.82 \times 10^{-5}\text{ m}^3$

$$\rho = \frac{0.512\text{ kg}}{6.82 \times 10^{-5}\text{ m}^3} = \mathbf{7.5 \times 10^3}\text{ kg m}^{-3}$$

(c) percentage uncertainties

- mass: $0.001/0.512 = 0.20\%$; x: $0.01/8.02 = 0.12\%$; y: $0.01/4.15 = 0.24\%$; z: $0.01/2.05 = 0.49\%$
- total $= 0.20 + 0.12 + 0.24 + 0.49 = \mathbf{1.1}\%$

(d) the **height** z

- it is the smallest length, so the same ± 0.01 cm is the largest percentage of it

(e) upthrust $= \rho_{\text{water}} g V$

$$F = (1000\text{ kg m}^{-3})(9.81\text{ m s}^{-2})(6.82 \times 10^{-5}\text{ m}^3) = \mathbf{0.67}\text{ N}$$

- the block's weight is $mg = (0.512\text{ kg})(9.81\text{ m s}^{-2}) = 5.0$ N, much larger, so it **sinks**

(f) floating: upthrust = weight $\Rightarrow \rho_{\text{water}} V_{\text{sub}} = \rho_{\text{wood}} V_{\text{total}}$

$$\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{650}{1000} = 0.65 \Rightarrow \mathbf{65}\%\text{ below the surface}$$

(g)

$$\Delta p = \rho g h = (1000\text{ kg m}^{-3})(9.81\text{ m s}^{-2})(2.0\text{ m}) = 1.96 \times 10^4\text{ Pa}$$

- $A = 25\text{ m} \times 10\text{ m} = 250\text{ m}^2$

$$F = pA = (1.96 \times 10^4\text{ Pa})(250\text{ m}^2) = \mathbf{4.9 \times 10^6}\text{ N}$$

- assumption: flat bottom (same depth everywhere); atmospheric pressure is **not** included