

4.2 Equilibrium of forces

Name: _____ Class: _____ Date: _____

Total: 30 marks

KEY WORDS equilibrium 平衡 • vector triangle 矢量三角形 • force 力 • moment 力矩
• principle of moments 力矩原理

Objective

Build the skills to answer exam questions on **equilibrium** 平衡 — the principle of moments, the two conditions for equilibrium, and the vector triangle.

You must be able to:

- state and apply the **principle of moments**
- use the **two conditions** for equilibrium (no resultant force, no resultant moment)
- use a closed **vector triangle** for three coplanar forces, with numbers
- draw** labelled force arrows on a diagram before calculating anything
- take moments about a **hinge** and find the force the hinge itself exerts

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

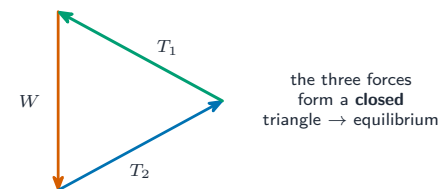
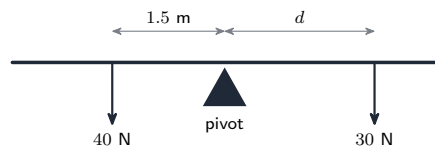
■ Conditions for equilibrium

A body is in **equilibrium** when **both**:

- the **resultant force** is zero, and
- the **resultant moment** (about any point) is zero.

■ Principle of moments

For a body not turning, the **total clockwise moment** = **total anticlockwise moment** about any point.



A closed vector triangle shows equilibrium

Worked example. Beam pivoted at its centre (so its weight gives no moment), 40 N at 1.5 m on the left:

$$40 \times 1.5 = 30 \times d \Rightarrow d = \frac{60}{30} = 2.0 \text{ m.}$$

Tip: take moments about the point where an **unknown** force acts, so it drops out.

■ Vector triangle

Three coplanar forces in equilibrium form a **closed triangle** (tip-to-tail). Solve it with the sine/cosine rule, or resolve into components and set $\sum F_x = 0$, $\sum F_y = 0$.

Worked example. A 250 N sign hangs from a horizontal wire and a wire at 40° to the vertical. Resolving the sloping tension T :

$$\text{vertical: } T \cos 40^\circ = 250 \Rightarrow T = \frac{250}{0.766} = 326 \text{ N}$$

$$\text{horizontal: } T \sin 40^\circ = F \Rightarrow F = 326 \times 0.643 = 210 \text{ N}$$

The vertical equation comes first because only **one** unknown appears in it. A closed triangle drawn to scale gives the same two answers.

■ Forces on a hinged beam

A beam hinged at one end and held up somewhere else has **three** forces on it, and the hinge one is the awkward one — so:

- Draw the arrows first**, labelled: the weight at the **centre of gravity**, the support force, the load.
- Take moments about the hinge**, because its own force then has zero moment and drops out.
- Get the hinge force **last**, from vertical equilibrium — and be ready for it to point **down**. If the support is between the hinge and the load, the hinge must pull the beam down to stop it tipping.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for a body to be in **equilibrium**. [2]

2.2 State the **principle of moments**. [1]

2.3 A 40 N weight hangs 1.5 m to the left of a pivot. How far to the **right** must a 30 N weight hang to balance it? [2]

2.4 State what a **closed vector triangle** tells you about three forces. [1]

3 Exam-style questions

3.1 Three coplanar forces act on an object that is in equilibrium. Which statement **must** be true? [1]

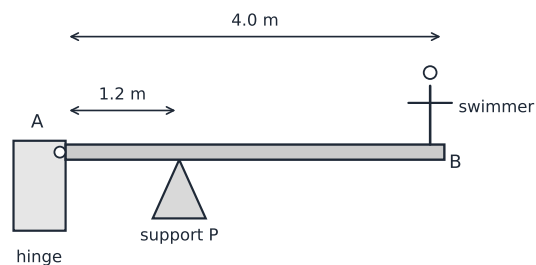
- **A** The three forces are equal in magnitude.
- **B** Drawn tip to tail, the three forces form a closed triangle.
- **C** The three forces all act through the same point and are parallel.
- **D** The resultant of two of the forces is equal in both magnitude and direction to the third.

3.2 A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot. Calculate the distance on the other side at which a 30 N weight balances the beam. [3]

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports **A** (left end) and **B** (right end). A 40 N load sits 1.0 m from **A**. **Show that** the upward force on support **B** is 40 N. [3]

3.4 Explain why, when balancing a **heavy uniform rod**, you must include its **weight** in the moments calculation. [2]

3.5 A uniform diving board AB has length 4.0 m and weight 800 N. It is hinged to a wall at end A and rests on a support P a distance 1.2 m from A. A swimmer of weight 620 N stands at end B, and the board is in equilibrium.



(a) **State** the principle of moments. [2]

(b) On the diagram above, **draw** arrows showing the **weight of the board**, the **force from the support P** and the **weight of the swimmer**. **Label** each arrow. [3]

(c) By taking moments about A, **determine** the force that the support P exerts on the board. [3]

(d) **Determine** the magnitude and the direction of the force that the hinge at A exerts on the board. [3]

(e) **Explain** why taking moments about A is a better choice than taking them about P. [1]

(f) The swimmer now walks slowly back towards A. **Give** the distance from A at which she must stand for the hinge to exert **no** force on the board. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **resultant force** is zero
- and** the **resultant moment** about any point is zero

2.2

- principle of moments: **total clockwise moment** about any point = **total anti-clockwise moment** about that point

2.3

- take moments about the pivot

$$(40 \text{ N})(1.5 \text{ m}) = (30 \text{ N})d \Rightarrow d = \mathbf{2.0 \text{ m}}$$

2.4

- the three forces are **in equilibrium** (resultant zero)

3.1 B

- in equilibrium the three forces have zero resultant, so tip-to-tail they close into a **triangle**
- D** is the trap: the resultant of two must be equal and **opposite** to the third, not in the same direction

3.2

- the beam's weight acts at the pivot, so it has **no moment**

$$(40 \text{ N})(1.5 \text{ m}) = (30 \text{ N})d$$

$$d = \frac{60 \text{ N m}}{30 \text{ N}} = \mathbf{2.0 \text{ m}}$$

3.3

- take moments about **A** so the force at A drops out

$$B \times 4.0 \text{ m} = (60 \text{ N})(2.0 \text{ m}) + (40 \text{ N})(1.0 \text{ m}) = 160 \text{ N m}$$

$$B = \frac{160 \text{ N m}}{4.0 \text{ m}} = \mathbf{40 \text{ N}}$$

3.4

- the rod's weight acts at its **centre of gravity**, a distance from the pivot
- so it contributes a **moment** that must be included

3.5

(a) for a body in **equilibrium**

- sum of **clockwise moments** about any point = sum of **anticlockwise moments** about that same point

(b)

- arrow **down** at the middle of the board (2.0 m from A) labelled *weight of board* / 800 N
- arrow **up** at P labelled *support force*
- arrow **down** at B labelled *weight of swimmer* / 620 N

(c) moments about A (hinge force has no moment)

- clockwise = $(800 \text{ N})(2.0 \text{ m}) + (620 \text{ N})(4.0 \text{ m}) = 4080 \text{ N m}$
- anticlockwise = $P \times 1.2 \text{ m}$

$$P = \frac{4080 \text{ N m}}{1.2 \text{ m}} = \mathbf{3400 \text{ N}}$$

(d) vertical equilibrium: up = down

- $3400 \text{ N} + F_{\text{hinge}} = 800 \text{ N} + 620 \text{ N}$
- $F_{\text{hinge}} = -1980 \text{ N}$, so **1980 N downwards**

(e) the hinge force is unknown; moments **about** A give it zero moment, so it drops out

(f) no hinge: take moments about **P**

- board's weight is $2.0 \text{ m} - 1.2 \text{ m} = 0.80 \text{ m}$ to the right of P
- clockwise: $(800 \text{ N})(0.80 \text{ m}) = 640 \text{ N m}$
- swimmer must be on the **other** side of P, a distance $(1.2 \text{ m} - x)$ from it

$$(620 \text{ N})(1.2 \text{ m} - x) = 640 \text{ N m} \Rightarrow x = \mathbf{0.17 \text{ m}}$$

from A