

4.2 Equilibrium of forces

Name: _____ Class: _____ Date: _____

Total: 15 marks

Objective

Build the skills to answer exam questions on **equilibrium** 平衡—the principle of moments, the two conditions for equilibrium, and the vector triangle.

You must be able to:

- state and apply the **principle of moments**
- use the **two conditions** for equilibrium (no resultant force, no resultant moment)
- use a closed **vector triangle** for three coplanar forces

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

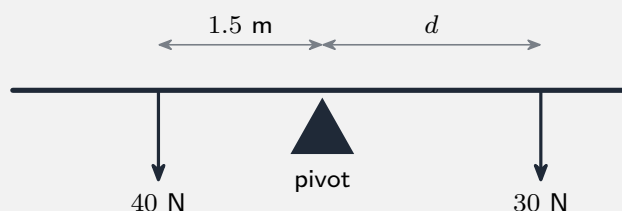
■ Conditions for equilibrium

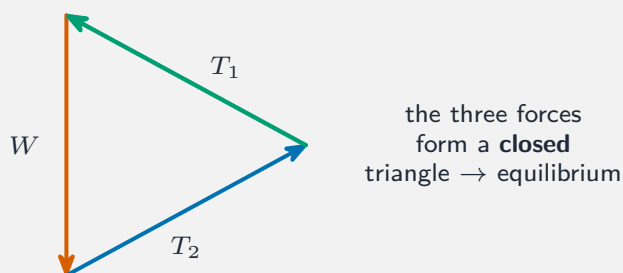
A body is in **equilibrium** when **both**:

1. the **resultant force** is zero, and
2. the **resultant moment** (about any point) is zero.

■ Principle of moments

For a body not turning, the **total clockwise moment** = **total anticlockwise moment** about any point.





A closed vector triangle shows equilibrium

Worked example. Beam pivoted at its centre (so its weight gives no moment), 40 N at 1.5 m on the left:

$$40 \times 1.5 = 30 \times d \Rightarrow d = \frac{60}{30} = 2.0 \text{ m.}$$

Tip: take moments about the point where an **unknown** force acts, so it drops out.

■ Vector triangle

Three coplanar forces in equilibrium form a **closed triangle** (tip-to-tail). Solve it with the sine/cosine rule, or resolve into components and set $\sum F_x = 0$, $\sum F_y = 0$.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for a body to be in **equilibrium**. [2]

2.2 State the **principle of moments**. [1]

2.3 A 40 N weight hangs 1.5 m to the left of a pivot. How far to the **right** must a 30 N weight hang to balance it? [2]

2.4 State what a **closed vector triangle** tells you about three forces. [1]

3 Exam-style questions

3.1 A 30 cm ruler balances on a fulcrum at the 15 cm mark. A 50 g mass is placed at 23 cm and a 20 g mass at 11 cm. Which mass at the 7 cm mark restores balance? [1]

- **A** 30 g
 - **B** 40 g
 - **C** 47 g
 - **D** 133 g
-

3.2 A uniform beam of length 4.0 m is pivoted at its centre. A 40 N weight hangs 1.5 m from the pivot. Calculate the distance on the other side at which a 30 N weight balances the beam. [3]

3.3 A uniform beam of weight 60 N and length 4.0 m rests on supports **A** (left end) and **B** (right end). A 40 N load sits 1.0 m from A. Calculate the upward force on support **B**. [3]

3.4 Explain why, when balancing a **heavy uniform rod**, you must include its **weight** in the moments calculation. [2]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **4.2 Equilibrium of forces** past-paper sheet in the Library, or try these in **Practice mode**:

- 9702/22 N25 —Q3 (equilibrium and moments)
- 9702/11 J25 —Q15 (principle of moments)
- 9702/23 J25 —Q2 (three forces in equilibrium)
- 9702/22 M25 —Q2 (equilibrium of a beam)

Solutions

2.1 The **resultant force** is zero **and** the **resultant moment** (about any point) is zero.

2.2 For a body in equilibrium, the **total clockwise moment** about any point equals the **total anticlockwise moment** about that point.

2.3 $40 \times 1.5 = 30 \times d \Rightarrow d = 2.0 \text{ m}$.

2.4 The three forces are **in equilibrium** (their resultant is zero).

3.1 B —40 g. Clockwise $50 \times 8 = 400$; anticlockwise $20 \times 4 + m \times 8 = 80 + 8m$; balance $\Rightarrow 8m = 320 \Rightarrow m = 40 \text{ g}$.

3.2 Beam's weight acts at the pivot (no moment); $40 \times 1.5 = 30 \times d$; $d = 60/30 = 2.0 \text{ m}$.

3.3 Take moments about **A**: $B \times 4.0 = 60 \times 2.0 + 40 \times 1.0 = 120 + 40 = 160$; $B = 160/4.0 = 40 \text{ N}$.

3.4 The rod's weight acts at its **centre of gravity**, at a distance from the pivot, so it contributes a **moment** that must be included or the balance equation is wrong.