A-LEVEL PHYSICS • EXERCISE SHEET

25.2 Stellar radii

Name: ____________ Class: ________ Date: __________ **Total: 26 marks**

KEY WORDS stefan–boltzmann law 斯特藩-玻尔兹曼定律 • wien's displacement law 维恩位移定律 • temperature 温度 • wavelength 波长 • blackbody 黑体

Objective

Build the skills to answer exam questions on **stellar temperature and radius**.

You must be able to:

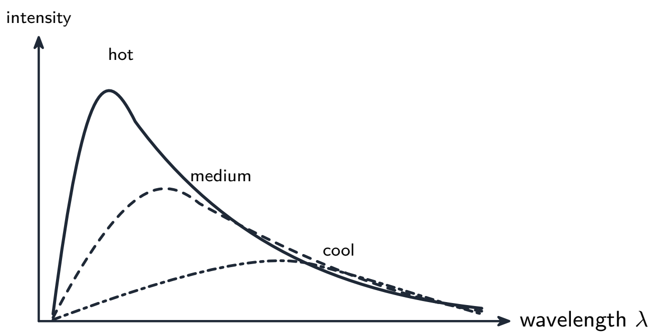
- use **Wien's law** $\lambda_{\max}T = \text{constant}$
- use the **Stefan–Boltzmann law** $L = 4\pi\sigma r^2 T^4$
- combine them to estimate a star's **radius**

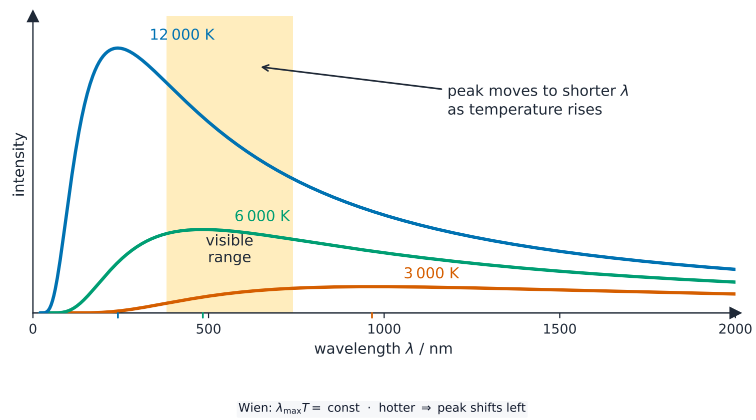
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Wien's law

A star radiates a **black-body** spectrum; a **hotter** star radiates more and peaks at a **shorter** wavelength:





A blackbody curve and Wien law

$$\lambda_{\max}T = b, \qquad b = 2.90 \times 10^{-3}\ \text{m K}.$$

■ Stefan–Boltzmann law and radius

$$L = 4\pi\sigma r^2 T^4, \qquad \sigma = 5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4}.$$

Find T from Wien's law, then $r = \sqrt{\dfrac{L}{4\pi\sigma T^4}}$.

Worked example. $\lambda_{\max} = 500$ nm: $T = \dfrac{2.90 \times 10^{-3}}{500 \times 10^{-9}} = 5800$ K.

2 Practice

Now apply the methods above.

2.1 Write **Wien's displacement law**. [1]

2.2 A star's spectrum peaks at 500 nm. Find its surface **temperature**. [2]

2.3 Write the **Stefan–Boltzmann law**. [1]

2.4 State how λ_{max} changes for a **hotter** star. [1]

3 Exam-style questions

3.1 A **hotter** star has its peak emission at a: [1]

- **A** longer wavelength
- **B** shorter wavelength
- **C** the same wavelength
- **D** no peak

3.2 A star's black-body spectrum peaks at 290 nm. Calculate its surface **temperature**. [2]

3.3 Star P has radius 7.0×10^8 m and surface temperature 5800 K. Star Q has the **same** surface temperature but **three times** the radius. ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.)

(a) Calculate the luminosity of star P. [2]

(b) Without further calculation, state and explain the luminosity of star Q. [2]

3.4 Outline how a star's **radius** can be estimated from its spectrum. [2]

3.5 The light from a distant star is analysed. Its intensity is greatest at a wavelength of 580 nm, and its radiant flux intensity at the Earth is 3.6×10^{-10} W m^{-2}. The star is 8.4×10^{17} m away. Take Wien's displacement constant as 2.90×10^{-3} m K and $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.

(a) **State** Wien's displacement law. [2]

(b) **Determine** the surface temperature of the star. [2]

(c) **Determine** the luminosity of the star. [2]

(d) **Determine** the radius of the star, and **compare** it with the Sun's radius of 7.0×10^8 m. [3]

(e) A second star has the **same** luminosity but a surface temperature twice as high. **Determine** the ratio $\dfrac{\text{radius of the second star}}{\text{radius of the first}}$, and **state** what kind of star that suggests.[3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\lambda_{\max}T = \text{constant}$ (or $\lambda_{\max} \propto 1/T$)

2.2

- $T = b/\lambda_{\max}$

$$T = \frac{2.90 \times 10^{-3}\ \text{m K}}{500 \times 10^{-9}\ \text{m}} = \mathbf{5800}\ \text{K}$$

2.3

- $L = 4\pi\sigma r^2 T^4$

2.4

- it is **shorter** ($\lambda_{\max} \propto 1/T$)

3.1 B

- a hotter star has its peak at a **shorter wavelength**

3.2

- $T = b/\lambda_{\max}$

$$T = \frac{2.90 \times 10^{-3}\ \text{m K}}{290 \times 10^{-9}\ \text{m}} = \mathbf{1.0 \times 10^4}\ \text{K}$$

3.3

(a) $L = 4\pi r^2 \sigma T^4$

$$L = 4\pi(7.0 \times 10^8\ \text{m})^2(5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4})(5800\ \text{K})^4 = \mathbf{4.0 \times 10^{26}}\ \text{W}$$

(b) $L \propto r^2$ at fixed temperature

- so tripling the radius makes the luminosity $3^2 = 9$ times larger: $\mathbf{3.6 \times 10^{27}}$ W

3.4

- find T from $\lambda_{\max}$ (Wien's law) and measure L
- then use the Stefan–Boltzmann law $L = 4\pi\sigma r^2 T^4$ to solve for r

3.5

(a) the wavelength at which the emitted intensity is a **maximum** is **inversely proportional** to the thermodynamic temperature of the body

- $\lambda_{\max}T = \text{constant}$

(b) $T = b/\lambda_{\max}$

$$T = \frac{2.90 \times 10^{-3}\ \text{m K}}{580 \times 10^{-9}\ \text{m}} = \mathbf{5.0 \times 10^3}\ \text{K}$$

(c) $L = 4\pi d^2 F$

$$L = 4\pi(8.4 \times 10^{17}\ \mathrm{m})^2(3.6 \times 10^{-10}\ \mathrm{W\ m^{-2}}) = \mathbf{3.2 \times 10^{27}}\ \mathrm{W}$$

(d) $r = \sqrt{L/(4\pi\sigma T^4)}$

$$r = \sqrt{\frac{3.19 \times 10^{27}\ \mathrm{W}}{4\pi(5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\,K^{-4}})(5000\ \mathrm{K})^4}} = \mathbf{2.7 \times 10^{9}}\ \mathrm{m}$$

- a few solar radii, so a star of ordinary main-sequence size

(e) at fixed luminosity r^2T^4 is constant, so $r \propto 1/T^2$

- doubling T gives a radius $1/4$ as large, i.e. a ratio of 0.25
- a star of the same luminosity packed into a quarter of the radius and much hotter suggests a **white dwarf** rather than a main-sequence star