A-LEVEL PHYSICS • EXERCISE SHEET

24.2.1 Intensity against hardness, attenuation through layers, and sharpness against contrast

Name: ____________ Class: ________ Date: __________ **Total: 35 marks**

KEY WORDS contrast 对比度 • x-ray tube X 射线管 • contrast medium 造影剂 • target 靶 • electron 电子

Objective

Build the skills to answer the **X-ray imaging** questions that sheet 24.2 does not reach. That sheet produces X-rays, uses $\lambda_{\text{min}} = hc/(eV)$ and $I = I_0 e^{-\mu x}$, defines contrast and describes a CT scan. Three things the topic is really assessed on are absent from it: how the tube's two controls set the **intensity** and the **hardness** 硬度 separately, what makes an image **sharp** as opposed to high in **contrast** 对比度, and the **half-value thickness** 半值厚度.

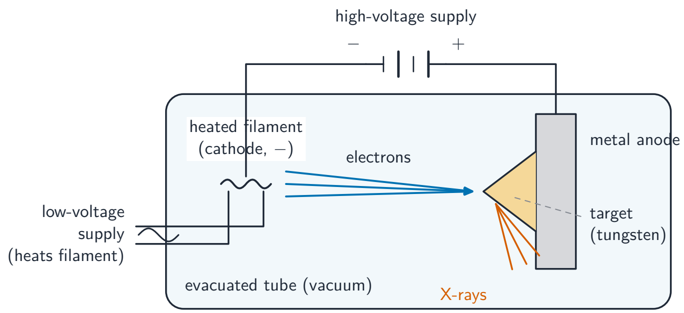
You must be able to:

- explain how X-rays are produced by electron bombardment, and calculate the minimum wavelength from the accelerating p.d.
- state which control sets the **intensity** of the beam and which sets its **hardness**, and why
- use $I = I_0 e^{-\mu x}$, including through **layers** of different materials, and use the half-value thickness
- distinguish **sharpness** from **contrast**, and state what improves each

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

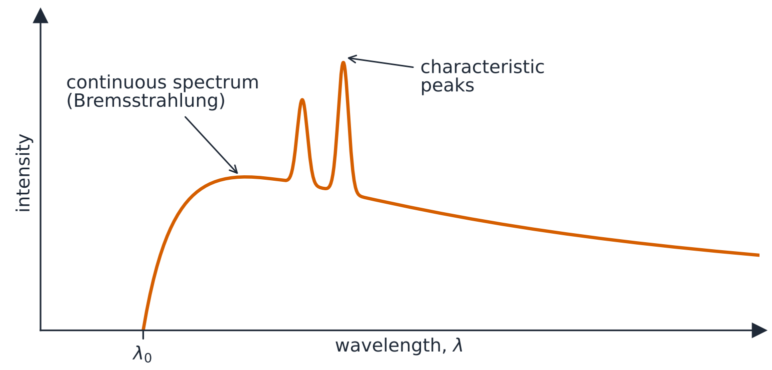
■ The tube has two controls, and they do different jobs



Electrons are released from a **heated filament**, accelerated through a large **p.d.**, and stopped suddenly by a metal **target**. Most of their kinetic energy becomes heat, which is why the target is cooled; a small fraction becomes X-ray photons.

Control	Sets	Why
filament current	the **intensity**	a hotter filament emits **more electrons per second**, so more photons per second
accelerating p.d.	the **hardness** (penetrating power)	a larger p.d. gives each electron more energy, so **higher-energy, shorter-wavelength** photons

- Swapping these two round is the commonest error in the topic. The p.d. does **not** set the intensity, and the filament current does **not** set the hardness.
- A metal **filter** is placed in the beam to absorb the softest X-rays, which would be absorbed in the patient's skin without ever reaching the detector, adding dose but no information.



X-ray: continuous bremsstrahlung + characteristic peaks

The spectrum stops abruptly at a **minimum wavelength**, which is the case where an electron gives up **all** its kinetic energy in a single collision:

$$eV = \frac{hc}{\lambda_{\text{min}}} \quad \Rightarrow \quad \lambda_{\text{min}} = \frac{hc}{eV}$$

Worked example. *Find λ_{min} for a tube at 50 kV.*

$$\lambda_{\text{min}} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{(1.60 \times 10^{-19})(50 \times 10^{3})} = \mathbf{2.5 \times 10^{-11}} \text{ m}$$

- λ_{min} depends **only** on the p.d., not on the target material or the current. Doubling the p.d. halves it.

Attenuation, layers, and the half-value thickness

$$I = I_0 e^{-\mu x}$$

Worked example. *A beam passes through 2.8 cm of a material with $\mu = 1.4$ cm^{-1}. What fraction is absorbed?*

$$\frac{I}{I_0} = e^{-1.4 \times 2.8} = e^{-3.92} = 0.020$$

So 2.0% is transmitted and **98**% is absorbed.

- Read the question twice: "absorbed" is $1 - I/I_0$, not I/I_0.
- Keep μ and x in the **same** units. A μ in cm^{-1} needs x in cm.

Through layers, the exponents **add**, because each layer attenuates what the previous one passed on:

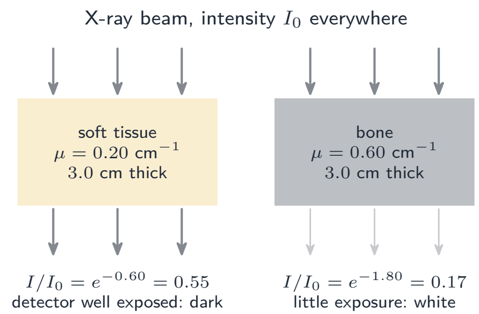
$$\frac{I}{I_0} = e^{-(\mu_1 x_1 + \mu_2 x_2)}$$

The **half-value thickness** is the thickness that halves the intensity:

$$x_{1/2} = \frac{\ln 2}{\mu}$$

Here $x_{1/2} = 0.693/1.4 = 0.50$ cm, so 2.8 cm is 5.6 half-value thicknesses and $(1/2)^{5.6} = 0.020$ – the same answer, and a useful check.

Sharpness and contrast are different things



contrast comes from the difference in μ: the two exposures differ by a factor of 3

- **Contrast** is the difference in **degree of blackening** between neighbouring regions. It comes from a difference in μ – bone attenuates far more than soft tissue, so it shows white against grey. Contrast is improved by choosing a p.d. that maximises the difference in attenuation, or by using a **contrast medium** such as barium, whose large μ makes a soft-tissue organ visible at all.
- **Sharpness** is how well **defined the edges** are. It is improved by a **small** X-ray source (a small spot on the target), a **stationary** patient, and the detector **close** to the patient.
- **The two are independent.** An image can be sharp with poor contrast, or high in contrast but blurred. A question naming one is not asking about the other.

2 Practice

Now use the methods above.

2.1 State which control of an X-ray tube sets the **intensity** of the beam, and which sets its **hardness**. [2]

2.2 Calculate the minimum wavelength of the X-rays from a tube operating at 80 kV. [3]

2.3 State what is meant by the **half-value thickness**, and write it in terms of μ. [2]

2.4 State the difference between the **sharpness** and the **contrast** of an X-ray image. [2]

2.5 State why a metal filter is placed in the beam leaving an X-ray tube. [2]

3 Exam-style questions

3.1 An X-ray beam passes through 2.8 cm of tissue for which $\mu = 1.4\ \text{cm}^{-1}$.

(a) Calculate the percentage of the incident intensity that is **absorbed**. [3]

(b) Calculate the half-value thickness of this tissue, and use it to check your answer to (a). [3]

3.2 A beam passes through 1.5 cm of bone ($\mu = 2.9\ \text{cm}^{-1}$) and then 4.0 cm of soft tissue ($\mu = 0.20\ \text{cm}^{-1}$).

Calculate the fraction of the incident intensity that emerges. [4]

3.3 Explain why bone appears white and soft tissue grey on an X-ray image, and state what is meant by the **contrast** of the image. [3]

3.4 State **two** changes to the apparatus that would improve the **sharpness** of an X-ray image, and explain why each works. [4]

3.5 Computed tomography (CT) scanning produces a 3D image of an **internal** structure: **multiple** X-ray images of the same section are taken from different angles and combined to **obtain** a 2D image of that section, and the process is repeated along an axis so that the 2D images of many sections can be combined.

(a) State **two** advantages of a CT image over a single X-ray image of the same body part. [2]

(b) State **one** disadvantage, and explain it. [2]

3.6 A radiographer increases the accelerating p.d. of an X-ray tube.

State the effect on the minimum wavelength, on the penetrating power of the beam, and on the intensity. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **filament current** sets the intensity: a hotter filament releases more electrons per second
- the **accelerating p.d.** sets the hardness: a larger p.d. gives each electron more energy and so produces higher-energy, more penetrating photons

2.2

- $\lambda_{\text{min}} = hc/(eV)$
- $= (6.63 \times 10^{-34})(3.00 \times 10^{8}) \div (1.60 \times 10^{-19} \times 80 \times 10^{3})$
- $= \mathbf{1.6 \times 10^{-11}}$ m

2.3

- the **thickness** of a material that reduces the intensity of the beam to **half** its incident value
- $x_{1/2} = \ln 2/\mu$, or $0.693/\mu$

2.4

- **sharpness** is how well **defined the edges** of structures are in the image
- **contrast** is the difference in **degree of blackening** between neighbouring regions, which comes from a difference in attenuation

2.5

- it absorbs the **softest** (lowest-energy, longest-wavelength) X-rays
- those would be absorbed in the patient's skin without reaching the detector, so they add to the radiation dose without contributing to the image

3.1

(a) $I/I_0 = e^{-\mu x} = e^{-1.4\times 2.8} = e^{-3.92}$

- $= 0.020$, so 2.0% is transmitted
- the percentage **absorbed** is $100 - 2.0 =$ **98**%
- quoting 2.0% answers the opposite question

(b) $x_{1/2} = \ln 2/\mu = 0.693/1.4 =$ **0.50** cm

- 2.8 cm is $2.8/0.50 = 5.6$ half-value thicknesses, and $(1/2)^{5.6} = 0.020$, which agrees with (a)

3.2

- the exponents add, because each layer attenuates what the one before it transmitted
- bone: $\mu x = 2.9 \times 1.5 = 4.35$
- tissue: $\mu x = 0.20 \times 4.0 = 0.80$, giving a total of 5.15

- $I/I_0 = e^{-5.15} = \mathbf{5.8 \times 10^{-3}}$
- multiplying the two thicknesses, or averaging the two μ values, is the error this question is set to catch

3.3

- bone has a much **larger attenuation coefficient** than soft tissue, so far less of the beam passes through it
- less radiation reaches the detector behind bone, so that region is **less blackened** and appears white, while more passes through soft tissue and blackens the image there
- the **contrast** is the difference in degree of blackening between neighbouring regions of the image

3.4 any two, each with its reason:

- use a **smaller** X-ray source – a smaller spot on the target – so that the edge of a structure casts a narrower penumbra
- keep the **patient still**, since any movement during the exposure blurs every edge
- place the **detector close to the patient**, which reduces the geometric spreading of the beam between the structure and the image
- a **contrast medium** is not accepted here: it improves contrast, not sharpness

3.5

(a) any two: a CT image shows a **cross-section**, so structures are not superimposed on one another as they are in a single X-ray

- it has much better **contrast between soft tissues**, because each voxel's attenuation is computed from many views rather than from one; and the image can be rotated and viewed as a three-dimensional structure or as any chosen slice

(b) the **radiation dose** is much higher

- because many exposures are taken from many angles for each section, and the process is repeated for every section along the axis; the scan also takes longer and the equipment is far more expensive

3.6

- the minimum wavelength **decreases**, since $\lambda_{\text{min}} = hc/(eV)$ and V has increased
- the beam becomes **more penetrating**, because the photons have higher energy: it is harder
- the **intensity** also rises somewhat, because each electron delivers more energy, but the intensity is **controlled** by the filament current, not by the p.d.