A-LEVEL PHYSICS • EXERCISE SHEET

24.1.1 Ultrasound scanning: pulse, echo and image

Name: ______________ Class: __________ Date: ____________ **Total: 27 marks**

KEY WORDS ultrasound 超声波 • specific acoustic impedance 声阻抗 • intensity reflection coefficient 强度反射系数 • transducer 换能器 • attenuation 衰减

Objective

Sheet 24.1 covered the piezo-electric effect and two substitutions. This sheet is the part the syllabus calls **diagnostic information**: how a pulse of **ultrasound 超声波** and its **echo 回声** give the depth of a boundary, why the **specific acoustic impedance 声阻抗** of the two tissues decides how much comes back, why a **coupling gel 耦合剂** is essential, how **attenuation 衰减** limits how deep a scan can see, and the frequency trade-off between resolution and penetration. This sheet covers syllabus 24.1.

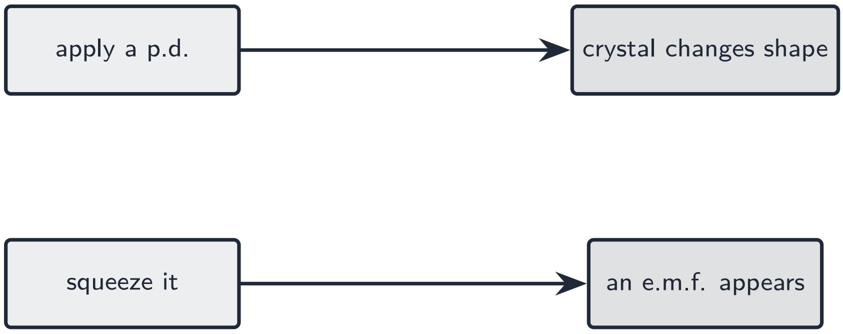
You must be able to:

- understand the **piezo-electric 压电** effect in both directions, and how ultrasound is generated and detected by a **piezoelectric transducer 换能器**
- understand how the reflection of **pulses** at boundaries between tissues gives diagnostic information about internal structures
- define specific acoustic impedance as $Z = \rho c$
- use $\dfrac{I_\mathrm{R}}{I_0} = \dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$ for the **intensity reflection coefficient 强度反射系数**
- recall and use $I = I_0 e^{-\mu x}$ for the attenuation of ultrasound in matter

1 Worked examples

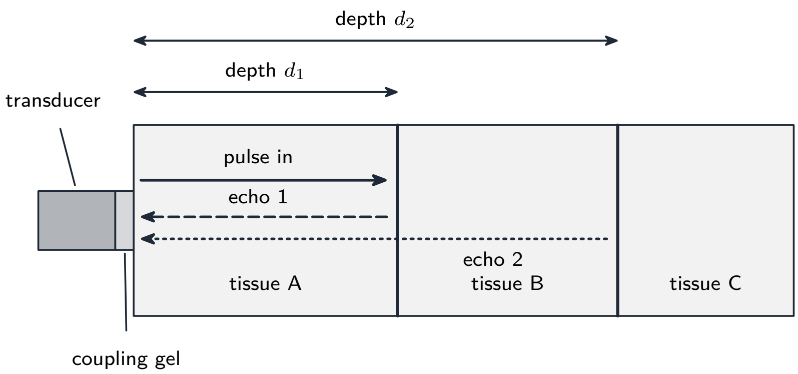
Study these first. Each one shows a **method** that a question later on this sheet needs.

> ■ **One crystal, two jobs**



- **Generating:** apply an **alternating p.d.** across the crystal. It changes shape in step with the p.d., so it **vibrates** at that frequency and sends out an ultrasound wave. Driving it at its **natural frequency** gives the largest amplitude, by resonance.
- **Detecting:** a returning ultrasound wave **changes the crystal's shape**, and a crystal whose shape changes generates an **e.m.f.** across its faces. That e.m.f. is the signal.
- So the **same** transducer does both, which is why ultrasound is sent as **short pulses**: the crystal transmits for a moment, then listens for the echoes before the next pulse. Send continuously and the returning echo cannot be distinguished from the outgoing wave.

■ From an echo time to a depth



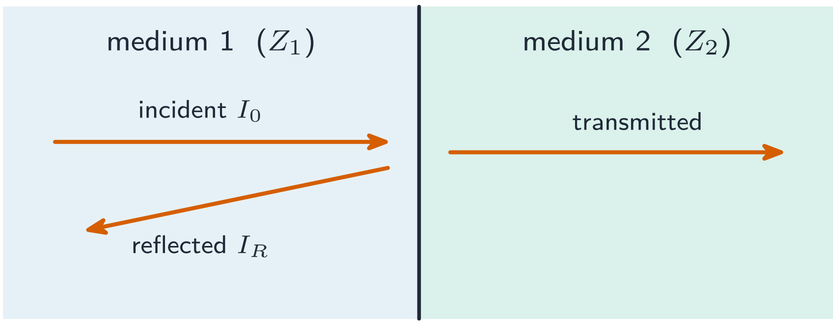
Each echo arrives after the pulse has travelled to that boundary **and back**, so a boundary at depth d returns its echo after $t = \frac{2d}{c}$, and $d = \frac{ct}{2}$. Forgetting the **2** is the standard error, and it halves every answer. The **coupling gel** matches the acoustic impedances of the transducer and the skin, so the pulse is not almost entirely reflected at the air gap.

Each boundary between two tissues reflects part of the pulse. The **time** the echo takes to come back gives the **depth**:

$$t = \frac{2d}{c} \qquad \Longrightarrow \qquad d = \frac{ct}{2}$$

- The **2 is the whole point**: the pulse travels **to** the boundary and **back**. Leaving it out halves every answer.
- c is the speed of ultrasound **in that tissue**, about 1540 m s^{-1} in soft tissue.
- A single line of echoes against time is an **A-scan**. Sweeping the transducer across the body and turning each echo into a bright dot whose brightness is the echo strength builds a two-dimensional **B-scan** – the familiar image.

■ How much comes back, and why the gel matters

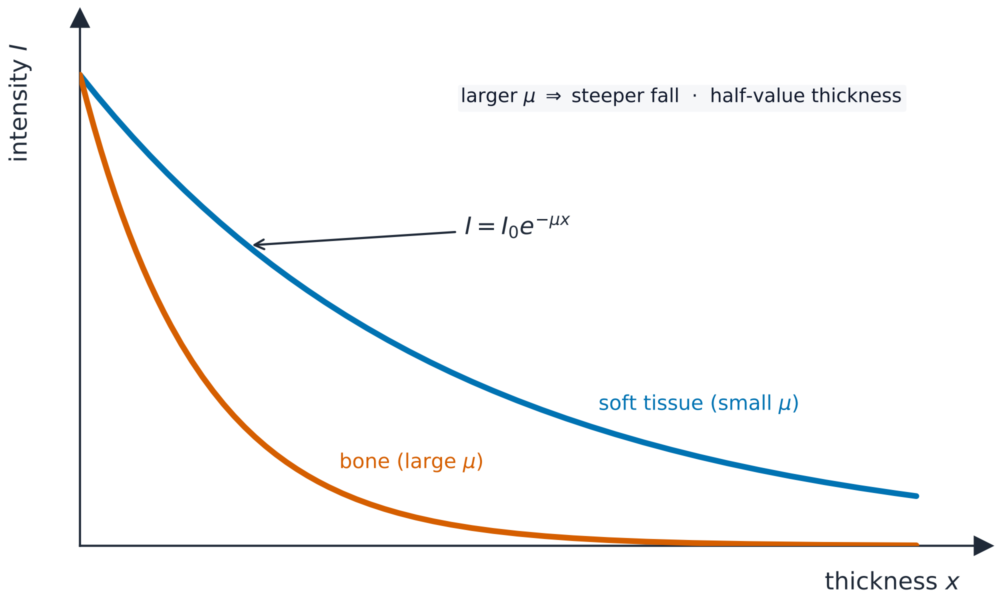


$$\frac{I_R}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$ — a bigger impedance difference gives a bigger echo

$$Z = \rho c \qquad \text{and} \qquad \frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

- Z is the **specific acoustic impedance**: the density of the medium multiplied by the speed of sound in it.
- The fraction reflected depends only on **how different** the two impedances are. Similar impedances reflect **little** (most is transmitted, and the pulse goes deeper); very different impedances reflect **almost everything**.
- Soft tissue and bone differ enough to give a strong echo, which is why bone shows up clearly. Two soft tissues differ only slightly, so most of the pulse continues – which is what lets the scan see past them.
- **Air is the problem.** Z for air is about 4×10^2 against 1.6×10^6 for tissue, so a tissue-air boundary reflects **more than 99.9%**. A thin air gap between the transducer and the skin would stop the scan working at all. The **coupling gel** has an impedance close to that of skin, so it **excludes the air** and lets the pulse in.

■ Attenuation, and choosing the frequency



$$I = I_0 e^{-\mu x}$$

- μ is the **attenuation coefficient** of the medium, in m^{-1}, so x must be in **metres**.
- For an **echo**, the pulse is attenuated **on the way there and on the way back**, so use the **total path length** $2d$, not d.
- **The frequency trade-off.** A higher frequency means a **shorter wavelength**, so smaller structures can be resolved – but μ **increases** with frequency, so the pulse is absorbed within a shorter distance. A scan of something deep uses a **lower** frequency and accepts poorer detail.
- Ultrasound is **non-ionising**, which is why it is used for a fetus where X-rays would be unacceptable.

2 Practice

Now use the methods above.

2.1 Explain how a piezo-electric transducer generates ultrasound, and how the **same** crystal detects it. [3]

2.2 Explain why ultrasound is sent into the body as short **pulses** rather than continuously. [2]

2.3 An echo returns 52 μs after the pulse is transmitted. The speed of ultrasound in the tissue is 1540 m s^{-1}. Calculate the depth of the boundary. [3]

2.4 Explain why almost no ultrasound would enter the body if there were a thin layer of air between the transducer and the skin. [3]

3 Exam-style questions

3.1 For a **strong** reflection of ultrasound at a boundary between two media, their specific acoustic impedances must be [1]

A	B
C	D

A equal to each other.
B very different from each other.
C both large.
D both small.

3.2 The specific acoustic impedances of three media, in kg m^{-2} s^{-1}, are: soft tissue 1.63×10^6, bone 6.40×10^6, air 4.29×10^2.

(a) Define the specific acoustic impedance of a medium. [1]

(b) Calculate the intensity reflection coefficient at a soft tissue-bone boundary. [2]

(c) Calculate the intensity reflection coefficient at a soft tissue-air boundary, and state what your answer means for ultrasound scanning. [3]

3.3 Ultrasound of initial intensity I_0 travels through soft tissue for which the attenuation coefficient is 20 m^{-1}.

(a) Calculate the fraction of the intensity remaining after the pulse has travelled 6.0 cm into the tissue. [3]

(b) The pulse reflects from a boundary at that depth and returns to the transducer. Calculate the fraction of the original intensity that arrives back, ignoring the loss at the boundary itself. [2]

(c) Explain why using a higher frequency improves the resolution of a scan but reduces the depth that can be imaged. [2]

(d) State and explain one advantage of using ultrasound rather than X-rays to image a fetus. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- an **alternating p.d.** is applied across the crystal, which makes it change shape in step with the p.d. and so **vibrate**
- the vibration sends out an ultrasound wave; driving the crystal at its natural frequency gives the largest amplitude
- a returning wave **changes the crystal's shape**, and a crystal whose shape changes generates an **e.m.f.**, which is the detected signal

2.2

- the same crystal both transmits and receives, so it must be **silent** while it listens
- with a continuous wave the returning echo could not be separated from the outgoing wave, and no time of flight – and therefore no depth – could be measured

2.3

- the pulse travels to the boundary **and back**, so the distance travelled is $2d$

$$d = \frac{ct}{2} = \frac{1540\ \text{m s}^{-1} \times 52 \times 10^{-6}\ \text{s}}{2}$$

$$d = \mathbf{4.0 \times 10^{-2}}\ \text{m (4.0 cm)}$$

2.4

- the specific acoustic impedance of air is about 4×10^2 kg m^{-2} s^{-1}, thousands of times smaller than that of tissue
- the reflection coefficient $\dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$ is then almost exactly 1
- so almost all the intensity is **reflected at the air gap** and essentially none enters the body; the coupling gel excludes the air for this reason

3.1 B

- the reflection coefficient depends on $(Z_1 - Z_2)^2$, so it is large only when the two impedances are very **different**
- equal impedances give zero reflection; the absolute sizes do not matter on their own

3.2

(a)

- the **density** of the medium multiplied by the **speed of sound** in it, $Z = \rho c$

(b)

- substitute into the reflection coefficient

$$\frac{I_R}{I_0} = \frac{(6.40 - 1.63)^2}{(6.40 + 1.63)^2} = \frac{(4.77)^2}{(8.03)^2} = \mathbf{0.35}$$

- the powers of ten cancel, so they need not be written out

(c)

- substitute again, this time keeping the powers of ten

$$\frac{I_R}{I_0} = \frac{(1.63 \times 10^6 - 4.29 \times 10^2)^2}{(1.63 \times 10^6 + 4.29 \times 10^2)^2} = \mathbf{0.999}$$

- more than 99.9% of the intensity is reflected at a tissue-air boundary, so a scan is impossible through even a thin layer of air – hence the coupling gel

3.3

(a)

- use $I = I_0 e^{-\mu x}$ with x in metres

$$\frac{I}{I_0} = e^{-20 \text{ m}^{-1} \times 0.060 \text{ m}} = e^{-1.2}$$

$$\frac{I}{I_0} = \mathbf{0.30}$$

(b)

- the total path is **there and back**, 2×6.0 cm $= 12$ cm

$$\frac{I}{I_0} = e^{-20 \text{ m}^{-1} \times 0.120 \text{ m}} = e^{-2.4} = \mathbf{0.091}$$

(c)

- a higher frequency has a **shorter wavelength**, so smaller structures can be distinguished and the **resolution** is better
- but the attenuation coefficient **increases** with frequency, so the pulse is absorbed within a shorter distance and less of it returns from deep boundaries

(d)

- ultrasound is **non-ionising**, whereas X-rays are **ionising radiation**
- ionising radiation can damage or mutate the rapidly dividing cells of a developing fetus, so ultrasound is far safer for this use