A-LEVEL PHYSICS • EXERCISE SHEET

23.2 Radioactive decay

Name: ____________ Class: ________ Date: __________ **Total: 32 marks**

KEY WORDS activity 活度 • half-life 半衰期 • spontaneous 自发 • decay constant 衰变常数 • random 随机

Objective

Build the skills to answer exam questions on **radioactive decay**.

You must be able to:

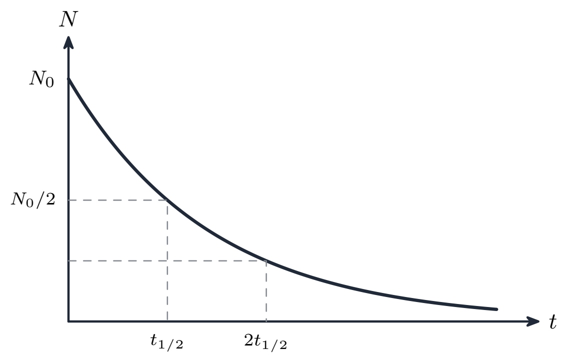
- describe decay as **spontaneous** and **random**
- use **activity** $A = \lambda N$ and $\lambda = 0.693/t_{1/2}$
- use the exponential decay $x = x_0 e^{-\lambda t}$ and **half-life**

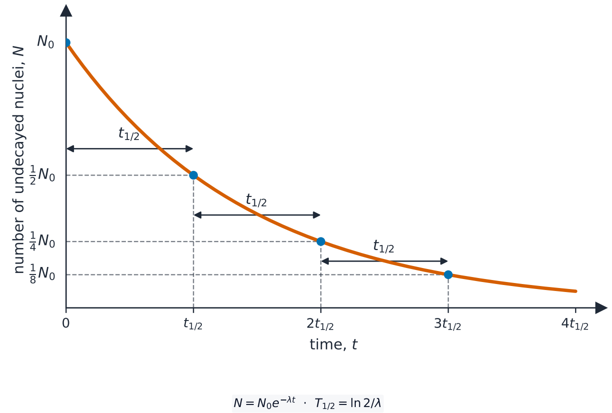
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Random and exponential

Decay is **spontaneous** (no trigger) and **random** (the count rate fluctuates). The number of undecayed nuclei falls **exponentially**, halving every **half-life**:





The exponential radioactive decay curve

$$A = \lambda N, \qquad \lambda = \frac{0.693}{t_{1/2}}, \qquad N = N_0 e^{-\lambda t}.$$

- **activity** A — decays per second (becquerel, Bq); **decay constant** λ — probability of decay per second.

The same exponential law applies to the **activity** and the **count rate**: $A = A_0 e^{-\lambda t}$.

■ Finding the time — take logs

For a **whole number** of half-lives, just halve repeatedly (after n half-lives a fraction $(\frac{1}{2})^n$ remains). For **any** time or fraction, take the natural log of $N = N_0 e^{-\lambda t}$:

$$\ln\frac{N}{N_0} = -\lambda t \quad \Longrightarrow \quad t = -\frac{1}{\lambda}\ln\frac{N}{N_0}.$$

Worked example. A source has half-life 8.0 days, so $\lambda = \dfrac{0.693}{8.0} = 0.0866\ \text{day}^{-1}$.

The time for its activity to fall to 20% is $t = -\dfrac{1}{0.0866}\ln(0.20) = 18.6$ days.

2 Practice

Now apply the methods above.

2.1 State **two** features of radioactive decay. [2]

2.2 Define **activity** and the **decay constant**. [2]

2.3 Define **half-life**. [1]

2.4 Write the equation linking **activity**, **decay constant** and **number** of nuclei. [1]

3 Exam-style questions

3.1 The **half-life** is the time for: [1]

- **A** all the nuclei to decay
- **B** half the (undecayed) nuclei to decay
- **C** the activity to reach exactly zero
- **D** the decay constant to halve

3.2 A source has a decay constant of 0.040 s^{-1}. Calculate its **half-life**. [2]

3.3 A sample has 2.0×10^{20} undecayed nuclei and a decay constant of 1.5×10^{-9} s^{-1}. Calculate its **activity**. [2]

3.4 A radioactive isotope has a half-life of 6.0 hours. What **fraction** of the original nuclei remains after 18 hours? [2]

3.5 A radioactive source has a half-life of 8.0 days. Calculate the **time** for its activity to fall to 20% of its initial value. [4]

3.6 Oxygen-15 is used as a tracer in medical imaging. It decays by β^+ emission with a half-life of 122 s. A freshly prepared sample contains 4.8×10^{14} nuclei.

(a) **State** what is meant by a **tracer**. [2]

(b) The decay is ${}^{15}_{8}\mathrm{O} \rightarrow {}^{P}_{Q}\mathrm{N} + {}^{R}_{S}\mathrm{Z}$. **State** the values of P, Q, R and S, and **name** the particle Z. [3]

(c) **Define** the **activity** of a radioactive sample. [1]

(d) **Determine** the decay constant of oxygen-15. Give a **unit** with your answer. [2]

(e) **Determine** the initial activity of the sample. [2]

(f) **Determine** the activity 10 minutes after preparation. [3]

(g) **Suggest** why a tracer with a half-life of a few **days** would be unsuitable for this use. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- it is **spontaneous** (unaffected by external conditions)
- and **random** (cannot predict which nucleus decays when)

2.2

- **activity** — the number of decays per unit time
- **decay constant** — the probability per unit time that a nucleus decays

2.3

- the time for **half** the undecayed nuclei (or the activity) to decay

2.4

- $A = \lambda N$

3.1 B

- half-life is the time for **half the undecayed nuclei to decay**

3.2

- $t_{1/2} = 0.693/\lambda$

$$t_{1/2} = \frac{0.693}{0.040\ \text{s}^{-1}} = \mathbf{17}\ \text{s}$$

3.3

- $A = \lambda N$

$$A = (1.5 \times 10^{-9}\ \text{s}^{-1})(2.0 \times 10^{20}) = \mathbf{3.0 \times 10^{11}}\ \text{Bq}$$

3.4

- 18 h = 3 half-lives
- fraction = $(1/2)^3 = \mathbf{1/8}$

3.5

- $\lambda = 0.693/t_{1/2}$

$$\lambda = \frac{0.693}{8.0\ \text{day}} = 0.0866\ \text{day}^{-1}$$

- $t = -(1/\lambda)\ln(A/A_0)$

$$t = -\frac{1}{0.0866\ \text{day}^{-1}}\ln(0.20) = \mathbf{18.6}\ \text{days}$$

3.6

(a) a radioactive substance introduced into a system

- whose progress can be followed from outside by detecting the radiation it emits

(b) $P = 15$, $Q = 7$, $R = 0$, $S = +1$

- Z is a **positron**

(c) the **rate of decay** — the number of nuclei decaying per unit time

(d) $\lambda = \ln 2 / t_{1/2}$

$$\lambda = \frac{0.693}{122\ \text{s}} = \mathbf{5.7 \times 10^{-3}}\ \text{s}^{-1}$$

(e) $A = \lambda N$

$$A = (5.68 \times 10^{-3}\ \text{s}^{-1})(4.8 \times 10^{14}) = \mathbf{2.7 \times 10^{12}}\ \text{Bq}$$

(f) 10 minutes is 600 s

$$A = A_0 e^{-\lambda t} = (2.73 \times 10^{12}\ \text{Bq})\, e^{-(5.68\times10^{-3}\ \text{s}^{-1})(600\ \text{s})} = \mathbf{9.1 \times 10^{10}}\ \text{Bq}$$

- about five half-lives, so roughly 1/32 of the start

(g) the patient would keep receiving **ionising radiation** long after the scan is over, raising the dose and the risk of cell damage

- the tracer should decay away soon after the measurement, which a half-life of minutes achieves and one of days does not