

23.2.1 Random decay, the exponential law and the log graph

Name: _____ Class: _____ Date: _____ Total: 29 marks

KEY WORDS radioactive decay 放射性衰变 • half-life 半衰期 • activity 放射性活度
• spontaneous 自发的 • random 随机的

Objective

Sheet 23.2 drilled the definitions and the standard half-life substitutions. This sheet is the rest of 23.2, and it is what the long questions are built from: what **spontaneous** 自发的 and **random** 随机的 actually mean and the evidence for them, the **exponential** 指数 law and the three different quantities it applies to, and – above all – the **log graph**, because $\ln A$ against t is a straight line whose gradient hands you λ directly. This sheet covers syllabus 23.2.

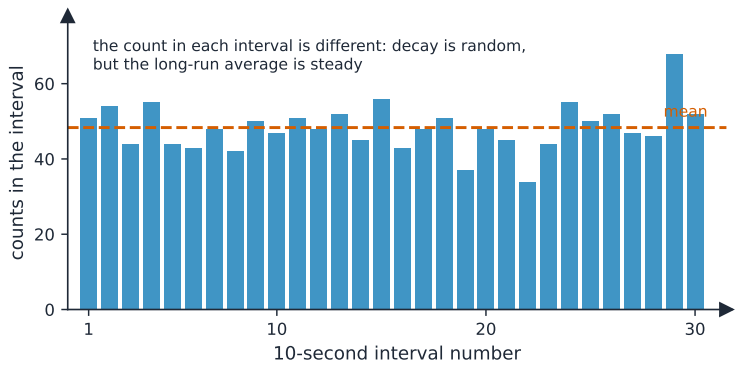
You must be able to:

- understand that fluctuations in **count rate** 计数率 are evidence for the random nature of **radioactive decay** 放射性衰变
- understand that radioactive decay is both spontaneous and random
- define **activity** 放射性活度 and the **decay constant** 衰变常数, and recall and use $A = \lambda N$
- define **half-life** 半衰期 and use $\lambda = \frac{0.693}{t_{1/2}}$
- understand the exponential nature of radioactive decay, and sketch and use $x = x_0 e^{-\lambda t}$, where x may be the activity, the number of undecayed nuclei, or a received count rate

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

- Two words, two separate marks



word	what it means	how you would answer
spontaneous	the decay is not affected by anything outside the nucleus	changing the temperature, the pressure or the chemical state of the sample does not change the rate
random	you cannot predict which nucleus will decay next, or when; every undecayed nucleus has the same probability of decaying in the next interval	repeated counts over equal intervals fluctuate about a mean instead of repeating exactly

- The two words are independent, and a question asking for both gets no credit for saying the same thing twice.
- The fluctuation evidence is the one experimental fact you are expected to quote: the count rate from a long-lived source is **not constant**, even though its activity is.

■ The exponential law, and the three things it describes

$$x = x_0 e^{-\lambda t}$$

x may be any of three quantities, and they all decay with the **same** λ :

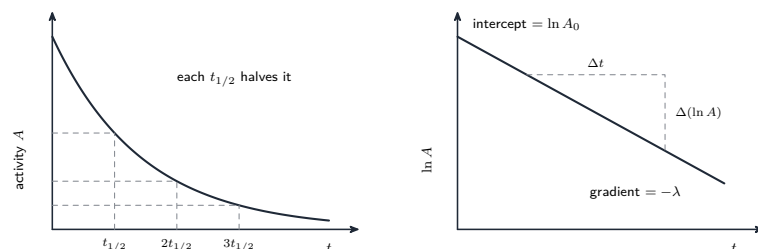
x	what it is
N	the number of undecayed nuclei left in the sample
$A = \lambda N$	the activity : decays per second, in becquerel
C	the received count rate at a detector

- The **received count rate is not the activity**. A detector collects only a small fraction of the radiation – it subtends a small solid angle, some radiation is ab-

sorbed on the way, and the detector is not 100% efficient – so C is proportional to A , not equal to it. Because it is *proportional*, it decays with the same λ and gives the same half-life.

- **Background radiation must be subtracted first.** A background count is a constant added to every reading; it does not decay, so leaving it in bends the graph.

■ The log graph – the workhorse of this topic



Take natural logs of $A = A_0 e^{-\lambda t}$ and you get $\ln A = \ln A_0 - \lambda t$. A **straight line** is then the evidence that the decay really is exponential, its **gradient** is $-\lambda$, and its **intercept** is $\ln A_0$. The same works for N and for a received count rate.

Take natural logs of both sides:

$$\ln A = \ln A_0 - \lambda t$$

That is $y = c + mx$, so:

1. a **straight line** is the evidence that the decay really is **exponential**;
2. the **gradient** is $-\lambda$, so $\lambda = -\text{gradient}$, and it is positive;
3. the **intercept** on the $\ln A$ axis is $\ln A_0$, so $A_0 = e^{\text{intercept}}$.

Worked example. A graph of $\ln(C/s^{-1})$ against t is a straight line through $(0, 4.80)$ and $(600 \text{ s}, 3.00)$. Find the decay constant and the half-life.

- the gradient is the change in $\ln C$ over the change in t

$$\text{gradient} = \frac{3.00 - 4.80}{600 \text{ s} - 0} = -3.0 \times 10^{-3} \text{ s}^{-1}$$

- so $\lambda = 3.0 \times 10^{-3} \text{ s}^{-1}$, and

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{3.0 \times 10^{-3} \text{ s}^{-1}} = \mathbf{230 \text{ s}}$$

■ Two shortcuts worth having

- **Whole numbers of half-lives.** After n half-lives the fraction left is $(\frac{1}{2})^n$. So $\frac{1}{8}$ left means $n = 3$; falling from 800 to 100 means three halvings.
- **Any fraction at all.** Rearrange the exponential and take logs:

$$\frac{x}{x_0} = e^{-\lambda t} \implies t = -\frac{1}{\lambda} \ln\left(\frac{x}{x_0}\right)$$

Keep λ and t in the **same** time unit, and remember $\ln$ of a fraction is negative, which is what makes t positive.

2 Practice

Now use the methods above.

2.1 Explain what is meant by saying that radioactive decay is **spontaneous** and **random**. [2]

2.2 A student records the count rate from a long-lived source over several equal intervals and finds that the readings differ. Explain how this supports the idea that decay is random. [2]

2.3 A radioactive isotope has a half-life of 4.5 hours. Calculate its decay constant, in s^{-1} . [3]

2.4 A sample contains 6.0×10^{20} undecayed nuclei of an isotope whose decay constant is $2.2 \times 10^{-8} \text{ s}^{-1}$. Calculate its activity. [2]

2.5 Sketch a graph of $\ln A$ against t for a radioactive source, and state what its gradient represents. [2]

2.6 The activity of a source falls to $\frac{1}{8}$ of its initial value. State how many half-lives have passed. [1]

3 Exam-style questions

3.1 The activity of a source falls from 800 Bq to 100 Bq in 27 minutes. Its half-life is [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** 3.0 minutes
B 9.0 minutes
C 13.5 minutes
D 27 minutes

3.2 A student measures the count rate from a source whose half-life is many years, recording the number of counts in five successive 10 s intervals:

412 389 405 421 398

(a) The activity of this source is effectively constant over the measurement. Explain why the five readings are nevertheless different. [2]

(b) Calculate the mean count rate, in s^{-1} . [2]

(c) The detector receives and records only 1.2% of the radiation emitted by the source. Calculate the activity of the source. [2]

3.3 A student measured the count rate C from a radioactive source at intervals, subtracted the background count, and plotted $\ln(C/\text{s}^{-1})$ against time t . The points lay on a straight line passing through (0, 5.60) and (900 s, 3.44).

(a) Explain how the straight line shows that the decay is exponential. [2]

(b) Use the graph to calculate the decay constant of the source. [2]

(c) Calculate the half-life of the source. [2]

(d) Calculate the corrected count rate at $t = 0$. [2]

(e) Suggest and explain the effect on the shape of the graph if the student had **not** subtracted the background count. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- **spontaneous**: the decay is not affected by anything outside the nucleus, so changing the temperature, pressure or chemical state does not change the rate
- **random**: it is impossible to predict which nucleus will decay next or when, and every undecayed nucleus has the same probability of decaying in the next interval

2.2

- the source's activity is effectively constant, so any difference between readings cannot be a change in the source
- the readings **fluctuate about a mean**, which is what a random process produces; a predictable process would give the same count every time

2.3

- convert the half-life to seconds first

$$t_{1/2} = 4.5 \text{ hours} \times 3600 \text{ s hour}^{-1} = 1.62 \times 10^4 \text{ s}$$

- then

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1.62 \times 10^4 \text{ s}} = \mathbf{4.3 \times 10^{-5} \text{ s}^{-1}}$$

2.4

- use $A = \lambda N$

$$A = 2.2 \times 10^{-8} \text{ s}^{-1} \times 6.0 \times 10^{20} = \mathbf{1.3 \times 10^{13} \text{ Bq}}$$

2.5

- a **straight line** with a **negative** gradient, starting at $\ln A_0$ on the vertical axis
- the gradient is $-\lambda$, so the decay constant is minus the gradient

2.6

- $\frac{1}{8} = \left(\frac{1}{2}\right)^3$, so **3 half-lives**

3.1 B

- $800 \rightarrow 400 \rightarrow 200 \rightarrow 100$ is **three** halvings
- $27 \text{ min} \div 3 = 9.0 \text{ min}$. **A** divides by the wrong number of halvings; **C** assumes two; **D** assumes one

3.2

(a)

- decay is a **random** process, so the number of nuclei decaying in any 10 s interval is not fixed

- the counts therefore **fluctuate about a mean** value; the variation is in the counting, not in the source

(b)

- add the five readings and divide by five, then by the interval

$$\bar{C} = \frac{412 + 389 + 405 + 421 + 398}{5} = 405 \text{ counts per } 10 \text{ s} = \mathbf{40.5 \text{ s}^{-1}}$$

(c)

- the detector records a fixed fraction of the emitted radiation

$$A = \frac{40.5 \text{ s}^{-1}}{0.012} = \mathbf{3.4 \times 10^3 \text{ Bq}}$$

3.3

(a)

- taking logs of $C = C_0 e^{-\lambda t}$ gives $\ln C = \ln C_0 - \lambda t$
- this has the form $y = c + mx$, so $\ln C$ against t is a straight line **only if** the decay is exponential; the points lying on a line is therefore the evidence

(b)

- the gradient of the line is $-\lambda$

$$\text{gradient} = \frac{3.44 - 5.60}{900 \text{ s} - 0} = -2.4 \times 10^{-3} \text{ s}^{-1}$$

$$\lambda = \mathbf{2.4 \times 10^{-3} \text{ s}^{-1}}$$

(c)

- use $\lambda = \frac{0.693}{t_{1/2}}$

$$t_{1/2} = \frac{0.693}{2.4 \times 10^{-3} \text{ s}^{-1}} = \mathbf{290 \text{ s}}$$

(d)

- the intercept is $\ln C_0$, so take the exponential of it

$$C_0 = e^{5.60} = \mathbf{270 \text{ s}^{-1}}$$

(e)

- the background count is a **constant** added to every reading, and it does **not** decay
- so at long times the measured count rate tends towards the background instead of towards zero, $\ln C$ levels off, and the graph **curves away** from the straight line rather than continuing down