A-LEVEL PHYSICS • EXERCISE SHEET

23.1 Mass defect and nuclear binding energy

Name: ____________ Class: ________ Date: __________ **Total: 27 marks**

KEY WORDS energy 能量 • mass defect 质量亏损 • binding energy 结合能 • binding energy per nucleon 比结合能 • nucleon number 核子数

Objective

Build the skills to answer exam questions on **mass defect and binding energy**.

You must be able to:

- use $E = mc^2$ and balance **nuclear equations**
- define **mass defect** and **binding energy**
- use the **binding energy per nucleon** curve for fusion and fission

1 Worked examples

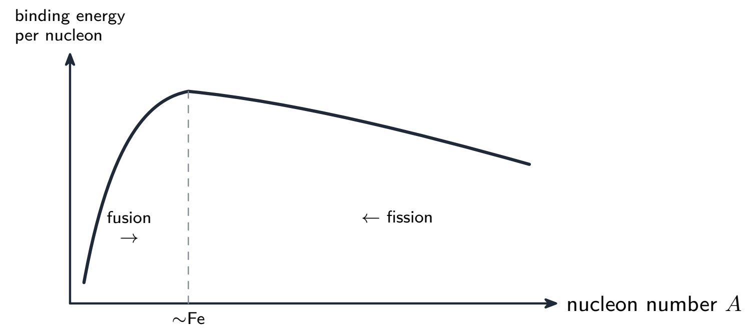
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

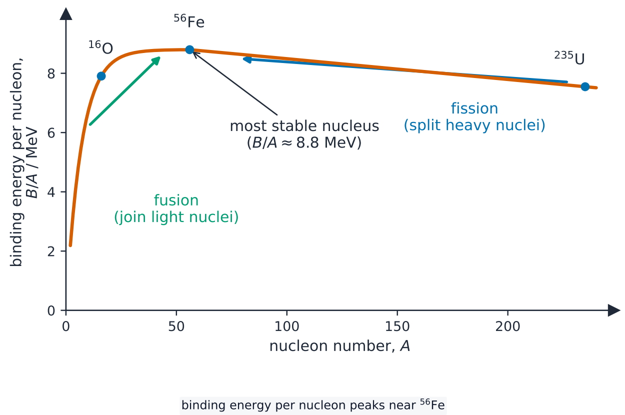
■ Mass-energy and binding energy

$$E = mc^2, \qquad \Delta E = c^2 \Delta m, \qquad 1\ \text{u} \approx 931\ \text{MeV}.$$

The **mass defect** is the difference between the mass of the separate nucleons and the mass of the nucleus; the **binding energy** is the energy equivalent of this ($\Delta E = c^2 \Delta m$) — the energy to split the nucleus apart.

■ Binding energy per nucleon





Binding energy per nucleon across the nuclides

The peak (near iron) is the **most stable**. **Fusion** of light nuclei and **fission** of heavy nuclei both move **towards the peak** — increasing binding energy per nucleon and **releasing energy**.

In nuclear equations, **nucleon number** and **charge** are conserved.

■ Energy released in a decay (from masses)

The energy released is the **mass lost** (parent mass − total product mass), converted with $1\ \text{u} \approx 931\ \text{MeV}$:

$$Q = \left(m_{\text{before}} - m_{\text{after}}\right) \times 931\ \text{MeV/u}.$$

Worked example. $^{238}_{92}\text{U} \rightarrow\ ^{234}_{90}\text{Th} +\ ^{4}_{2}\alpha$, with masses (u): U = 238.05079, Th

$= 234.04360$, $\alpha = 4.00260$. Mass lost $\Delta m = 238.05079 - (234.04360 + 4.00260) = 0.00459$ u; energy released $Q = 0.00459 \times 931 = 4.3$ MeV.

Working **backwards**: if the released energy is known, $\Delta m = Q/931$ u, which fixes a missing mass.

2 Practice

Now apply the methods above.

2.1 State **Einstein's** mass–energy equation. [1]

2.2 Define **mass defect**. [1]

2.3 Define **binding energy**. [1]

2.4 In ${}^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{A}_{8}\text{O} + {}^{1}_{1}\text{H}$, find the nucleon number A. [1]

3 Exam-style questions

3.1 The **most stable** nuclei have: [1]

- **A** the highest mass
- **B** the highest binding energy per nucleon
- **C** the lowest binding energy per nucleon
- **D** zero binding energy

3.2 A reaction has a mass defect of 0.030 u. Calculate the energy released. (1 u = 931 MeV.) [2]

3.3 Explain how the binding-energy-per-nucleon curve shows that **both** fusion of light nuclei **and** fission of heavy nuclei can release energy. [3]

3.4 Calculate the energy equivalent of a mass change of 1.0×10^{-29} kg. [2]

3.5 In the decay ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\alpha$, the masses (u) are: Po = 209.98286, Pb = 205.97444, $\alpha = 4.00260$. Calculate the energy released, in MeV. (1 u = 931 MeV.) [3]

3.6 Deuterium nuclei fuse to form helium-3 in the reaction

$${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$$

The masses are: deuterium 2.01410 u, helium-3 3.01603 u, neutron 1.00867 u. Take 1 u $= 1.66 \times 10^{-27}$ kg, $c = 3.00 \times 10^{8}$ m s^{-1} and 1 MeV $= 1.60 \times 10^{-13}$ J.

(a) **State** what is meant by the **mass defect** of a nucleus. [1]

(b) **Determine** the mass difference of this reaction, in u. [2]

(c) **Determine** the energy released, in joules and in MeV. [3]

(d) **Determine** the energy released, in MeV, when 1.0 g of deuterium is fused this way. [3]

(e) **Explain** why fusion releases energy for light nuclei but fission releases energy for heavy ones, referring to **binding energy per nucleon**. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $E = mc^2$

2.2

- the difference between the **total mass of the separate nucleons** and the **mass of the nucleus**

2.3

- the energy needed to **separate a nucleus** into its individual nucleons (equivalent to the mass defect)

2.4

- $14 + 4 = A + 1 \Rightarrow A = \mathbf{17}$

3.1 B

- the most stable nuclei have the **highest binding energy per nucleon**

3.2

- $E = \Delta m \times 931 \text{ MeV u}^{-1}$

$$E = (0.030 \text{ u})(931 \text{ MeV u}^{-1}) = \mathbf{28} \text{ MeV}$$

3.3

- both light nuclei (fusing) and heavy nuclei (splitting) move **towards the peak** of the curve
- this **increases** the binding energy per nucleon
- the products are more tightly bound, so energy is **released**

3.4

- $E = mc^2$

$$E = (1.0 \times 10^{-29} \text{ kg})(3.0 \times 10^{8} \text{ m s}^{-1})^2 = \mathbf{9.0 \times 10^{-13}} \text{ J}$$

3.5

- mass lost $\Delta m = 209.98286 \text{ u} - (205.97444 \text{ u} + 4.00260 \text{ u}) = 0.00582 \text{ u}$

$$Q = (0.00582 \text{ u})(931 \text{ MeV u}^{-1}) = \mathbf{5.4} \text{ MeV}$$

3.6

(a) the **difference** between the total mass of the separate nucleons and the mass of the assembled nucleus

(b) left: $2 \times 2.01410 \text{ u} = 4.02820 \text{ u}$; right: $3.01603 \text{ u} + 1.00867 \text{ u} = 4.02470 \text{ u}$

- difference = **0.00350** u

(c) $\Delta m = (0.00350 \text{ u})(1.66 \times 10^{-27} \text{ kg u}^{-1}) = 5.81 \times 10^{-30}$ kg

$$E = \Delta mc^2 = (5.81 \times 10^{-30} \text{ kg})(3.00 \times 10^8 \text{ m s}^{-1})^2 = 5.2 \times 10^{-13} \text{ J}$$

$$\frac{5.2 \times 10^{-13} \text{ J}}{1.60 \times 10^{-13} \text{ J MeV}^{-1}} = \mathbf{3.3} \text{ MeV}$$

(d) 1.0 g of deuterium contains

$$N = \frac{1.0 \times 10^{-3} \text{ kg}}{(2.01410 \text{ u})(1.66 \times 10^{-27} \text{ kg u}^{-1})} = 3.0 \times 10^{23}$$

nuclei

- the reaction consumes **two** nuclei per event, so there are 1.5×10^{23} events

$$\text{total} = (1.5 \times 10^{23})(3.3 \text{ MeV}) = \mathbf{4.9 \times 10^{23}} \text{ MeV}$$

(e) binding energy per nucleon rises steeply from hydrogen to a maximum near iron, then falls slowly for heavier nuclei

- energy is released whenever the products have a **greater** binding energy per nucleon than the reactants
- so joining light nuclei moves up the curve and releases energy, while splitting a heavy nucleus moves up it from the other side and does the same