23.1.1 The binding-energy curve, and the energy bookkeeping of a nuclear reaction

Name: ______________ Class: __________ Date: ____________ **Total: 31 marks**

KEY WORDS energy 能量 • binding energy 结合能 • nucleon number 核子数 • binding energy per nucleon 每核子结合能 • fusion 聚变

Objective

Sheet 23.1 gave you the definitions and two mass-defect substitutions. This sheet is the rest of 23.1, and it is where the long questions live: the **binding energy per nucleon** 每核子结合能 curve and what it actually tells you, deciding from the masses whether a reaction **releases** or **absorbs** energy, doing the arithmetic in both unit systems, and explaining why **fusion** 聚变 in a star needs an enormous temperature. This sheet covers syllabus 23.1.

You must be able to:

- understand the equivalence of mass and energy, $E = mc^2$, and use it
- represent nuclear reactions by **nuclear equations** and check that nucleon and proton number are conserved
- define and use **mass defect** 质量亏损 and **binding energy** 结合能
- sketch the variation of binding energy per nucleon with **nucleon number** 核子数
- explain what is meant by nuclear fusion and nuclear **fission** 裂变, and the relevance of binding energy per nucleon to both
- calculate the energy released in nuclear reactions using $E = c^2\Delta m$

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ The two definitions, and the trap in the second one

- The **mass defect** of a nucleus is the difference between the **total mass of its separate nucleons** and the **mass of the nucleus itself**. The nucleus is always the **lighter** of the two.
- The **binding energy** is the **energy needed to separate a nucleus into its individual nucleons** – equivalently, the energy released when those nucleons come together to form it.

△ Binding energy is **not** "energy stored in the nucleus". It is energy you would have to **put in**. Saying it is stored inside gets no credit, and it makes the sign of every later calculation wrong.

The two are the same thing in different units, linked by $E = mc^2$:

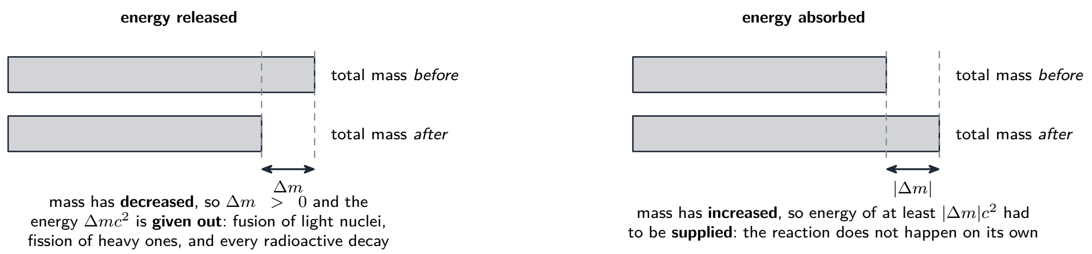
$$\text{binding energy} = (\text{mass defect}) \times c^2$$

■ Two unit systems, and when to use which

given in	use	typical answer
atomic mass units, u	$E/\text{MeV} = 931.5 \times \Delta m/\text{u}$	a few MeV per nucleon
kilograms	$E/\text{J} = \Delta m \times c^2$, with $c = 3.00 \times 10^8\ \text{m s}^{-1}$	around 10^{-12} J

- To go between them: 1 MeV $= 1.60 \times 10^{-13}$ J and 1 u $= 1.66 \times 10^{-27}$ kg.
- Keep **all five decimal places** of the masses until the very last line. The mass defect is a tiny difference between two nearly equal numbers, so rounding early destroys it – this is the single most common way marks are lost here.

■ Released or absorbed? Look at the sign



Always write Δm = (total mass before) − (total mass after) in that order. In atomic mass units, $E/\text{MeV} = 931.5 \times \Delta m/\text{u}$; in kilograms, $E/\text{J} = \Delta m \times c^2$. Quote the *sign* as well as the number.

$$\Delta m = (\text{total mass before}) - (\text{total mass after})$$

- $\Delta m > 0$: mass has been **lost**, and the energy Δmc^2 is **released**. Every spontaneous decay is like this, as are fusion of light nuclei and fission of heavy ones.
- $\Delta m < 0$: mass has been **gained**, so that much energy had to be **supplied** from outside. The reaction cannot happen on its own.

Worked example. In ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n$*, the masses in u are* ${}^2H = 2.01410$, ${}^3He = 3.01603$ *and* $n = 1.00867$. *Find the energy released.*

- check the equation balances: nucleon number $2 + 2 = 3 + 1$; proton number $1 + 1 = 2 + 0$. ✓

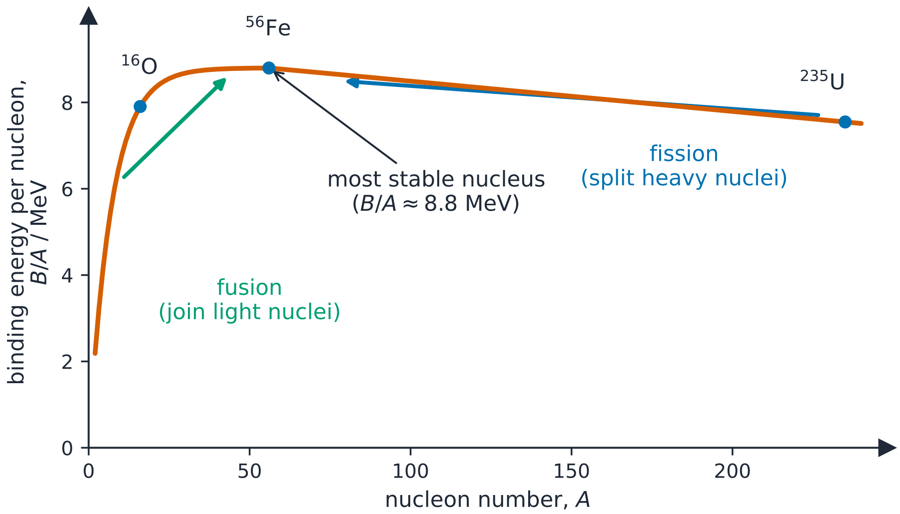
- before: $2 \times 2.01410\text{ u} = 4.02820\text{ u}$; after: $3.01603 + 1.00867 = 4.02470\text{ u}$

$$\Delta m = 4.02820\text{ u} - 4.02470\text{ u} = 3.50 \times 10^{-3}\text{ u}$$

- convert

$$E = 931.5\text{ MeV u}^{-1} \times 3.50 \times 10^{-3}\text{ u} = \mathbf{3.26}\text{ MeV, released}$$

The curve, and the one sentence that explains both fusion and fission



binding energy per nucleon peaks near ^{56}Fe

- The vertical axis is binding energy **per nucleon**, and **higher means more stable**.
- The maximum is at about **iron-56**, at roughly 8.8 MeV per nucleon: iron is the most stable nuclide, which is why no reaction releases energy by making anything beyond it.
- **Any reaction that moves nucleons UP the curve releases energy.** That single sentence is the answer to both halves:
  - **fusion** joins two **light** nuclei, far up the steep left-hand side, giving a much larger binding energy per nucleon;
  - **fission** splits a **heavy** nucleus, moving back up the gentle right-hand side towards iron.
- The rise on the left is **far steeper** than the fall on the right, which is why fusion releases several times more energy **per nucleon** than fission.

Why fusion needs a star

Two nuclei are both **positively charged**, so they **repel** each other electrostatically. To fuse, they must approach closely enough for the short-range **strong nuclear force** to take over, and that requires a very large kinetic energy to overcome the repulsion. A temperature of millions of kelvin is what gives the nuclei that kinetic energy – which is why fusion happens in stars and is so hard to sustain on Earth.

2 Practice

Now use the methods above.

2.1 Define the **mass defect** of a nucleus, and state which is larger: the mass of the nucleus, or the total mass of its separate nucleons. [2]

2.2 Define the **binding energy** of a nucleus. [2]

2.3 Calculate the energy, in MeV, equivalent to a mass of 1.00 u. Take 1 u = 1.66×10^{-27} kg, $c = 3.00 \times 10^{8}$ m s^{-1} and 1 MeV = 1.60×10^{-13} J. [3]

2.4 A nuclide of nucleon number 56 has a total binding energy of 492 MeV. Calculate its binding energy per nucleon. [2]

2.5 State what the position of the **maximum** of the binding-energy-per-nucleon curve tells you. [2]

3 Exam-style questions

3.1 Which statement about the binding energy of a nucleus is correct? [1]

A It is the energy stored inside the nucleus.
B It is the energy released when the nucleus decays.
C It is the energy needed to separate the nucleus into its individual nucleons.
D It is the energy equivalent of the total mass of the nucleus.

3.2 The stability of a nuclide is described by its binding energy per nucleon.

(a) Sketch the variation of binding energy per nucleon with nucleon number, from hydrogen to uranium. Mark the position of the maximum and give its approximate value. [3]

(b) Use your sketch to explain why energy is released **both** when light nuclei fuse **and** when heavy nuclei undergo fission. [3]

3.3 Deep inside the Sun, two helium-3 nuclei fuse:

$$^{3}_{2}\text{He} + ^{3}_{2}\text{He} \rightarrow\ ^{4}_{2}\text{He} + 2\,^{1}_{1}\text{H}$$

The masses, in u, are $^{3}\text{He} = 3.01603$, $^{4}\text{He} = 4.00260$ and $^{1}\text{H} = 1.00783$.

(a) Show that both nucleon number and proton number are conserved in this reaction. [2]

(b) Calculate the mass defect of the reaction, in u. [2]

(c) Calculate the energy released, in MeV. Take 1 u = 931.5 MeV. [2]

(d) Explain why this reaction takes place only at a very high temperature. [3]

(e) In a different nuclear reaction the total mass of the products is **greater** than the total mass of the reactants. State what this means about the energy of the reaction, and explain your answer. [2]

(f) Explain why fusing light nuclei releases much more energy **per nucleon** than the fission of a heavy nucleus. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the mass defect is the **difference** between the total mass of the separate nucleons and the mass of the nucleus they form
- the **total mass of the separate nucleons is larger**; the nucleus is always lighter

2.2

- the **energy needed to separate a nucleus into its individual nucleons**
- equivalently, the energy released when those separate nucleons come together to form the nucleus

2.3

- use $E = mc^2$ in SI units

$$E = 1.66 \times 10^{-27}\ \text{kg} \times (3.00 \times 10^8\ \text{m s}^{-1})^2 = 1.49 \times 10^{-10}\ \text{J}$$

- convert to MeV

$$E = \frac{1.49 \times 10^{-10}\ \text{J}}{1.60 \times 10^{-13}\ \text{J MeV}^{-1}} = \mathbf{934}\ \text{MeV}$$

- this is the tabulated 931.5 MeV, to the precision of the data given

2.4

- divide the total binding energy by the number of nucleons

$$\frac{492\ \text{MeV}}{56} = \mathbf{8.8}\ \text{MeV per nucleon}$$

2.5

- the maximum is at about **iron-56**, and it marks the **most stable** nuclide
- a higher binding energy per nucleon means more energy is needed to pull the nucleus apart; nothing beyond iron can be made with a release of energy

3.1 C

- binding energy is the energy that must be **put in** to separate the nucleons
- **A** reverses the direction; **B** is the energy released in a particular decay, not the binding energy; **D** would be about 10^5 times too large

3.2

(a)

- a **steep rise** from hydrogen through the light nuclei
- a **maximum** at about nucleon number **56**, at roughly 8.8 MeV per nucleon
- a **gentle fall** from there to about 7.6 MeV per nucleon at uranium

(b)

- energy is released whenever the products have a **higher binding energy per nucleon** than the reactants, that is, whenever the nucleons move **up** the curve
- **fusion** of two light nuclei moves far up the steep left-hand side, towards the maximum
- **fission** of a heavy nucleus produces two nuclei nearer the maximum, moving back up the gentle right-hand side

3.3

(a)

- nucleon number: $3 + 3 = 6$ on the left, and $4 + 2 \times 1 = 6$ on the right
- proton number: $2 + 2 = 4$ on the left, and $2 + 2 \times 1 = 4$ on the right

(b)

- total masses before and after, keeping all the decimal places

$$\text{before} = 2 \times 3.01603 \text{ u} = 6.03206 \text{ u}, \qquad \text{after} = 4.00260 + 2 \times 1.00783 = 6.01826 \text{ u}$$

$$\Delta m = 6.03206 \text{ u} - 6.01826 \text{ u} = \mathbf{1.380 \times 10^{-2}} \text{ u}$$

(c)

- multiply by the conversion factor

$$E = 931.5 \text{ MeV u}^{-1} \times 1.380 \times 10^{-2} \text{ u} = \mathbf{12.9} \text{ MeV}$$

(d)

- both nuclei carry a **positive charge**, so they **repel** each other electrostatically
- to fuse they must come close enough for the short-range **strong nuclear force** to act, which needs a very large kinetic energy to overcome that repulsion
- a very high temperature is what gives the nuclei kinetic energy of that size, since the mean kinetic energy of a particle is proportional to the thermodynamic temperature

(e)

- the reaction **absorbs** energy; it cannot happen spontaneously
- mass has increased, so by $E = \Delta mc^2$ energy of at least $|\Delta m|c^2$ must have been **supplied** from outside to create it

(f)

- the binding-energy-per-nucleon curve **rises very steeply** on the light side but **falls only gently** beyond the maximum
- so fusing two light nuclei moves each nucleon much further up the curve than splitting a heavy nucleus does, and the energy released per nucleon is correspondingly larger