A-LEVEL PHYSICS • EXERCISE SHEET

21.1 Characteristics of alternating currents

Name: ____________ Class: ________ Date: __________ **Total: 22 marks**

KEY WORDS period 周期 • power 功率 • frequency 频率 • peak value 峰值 • current 电流

Objective

Build the skills to answer exam questions on **alternating currents** — peak, r.m.s. and mean power.

You must be able to:

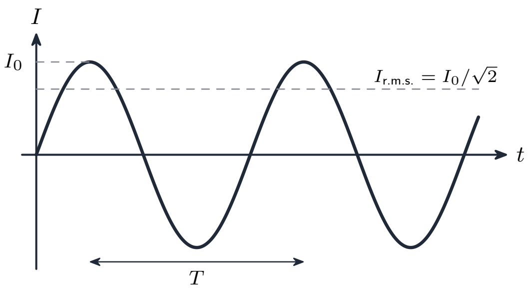
- use **period**, **frequency** and **peak value** for a.c.
- use $x = x_0 \sin \omega t$ and $I_{r.m.s.} = I_0/\sqrt{2}$
- use that the **mean power** is **half** the peak power

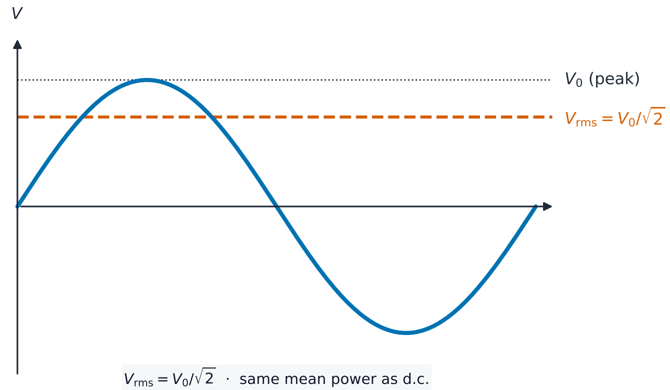
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Alternating current

An a.c. follows $I = I_0 \sin \omega t$:





RMS values of an alternating current

- **peak** I_0 (amplitude); **period** T; **frequency** $f = 1/T$; $\omega = 2\pi f$.

Reading the equation

Exam questions often **give** the a.c. as an equation, e.g. $V = 18\cos(40\pi t)$. Read off the **peak** ($V_0 = 18$ V) and the **angular frequency** ($\omega = 40\pi$ rad s^{-1}), then $T = \frac{2\pi}{\omega}$ and $f = \frac{1}{T}$.

Worked example. For $V = 18\cos(40\pi t)$: $\omega = 40\pi$, so $T = \frac{2\pi}{40\pi} = 0.050$ s and $f = 20$ Hz; $V_{r.m.s.} = 18/\sqrt{2} = 13$ V.

r.m.s. and mean power

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}}, \qquad V_{r.m.s.} = \frac{V_0}{\sqrt{2}}.$$

The **mean power** in a resistor is **half** the peak power; $\langle P \rangle = I_{r.m.s.} V_{r.m.s.} = \frac{V_{r.m.s.}^2}{R}$.

Worked example. $V_0 = 340$ V: $V_{r.m.s.} = 340/\sqrt{2} = 240$ V.

2 Practice

Now apply the methods above.

2.1 Define the **peak value** and the **period** of an alternating current. [2]

2.2 An a.c. has a peak current of 5.0 A. Find the **r.m.s.** current. [2]

2.3 State the relationship between the r.m.s. and peak **voltage**. [1]

2.4 Mains frequency is 50 Hz. Find the **period**. [1]

2.5 An a.c. voltage is $V = 18\cos(40\pi t)$. State the **peak voltage** and the **angular frequency**. [2]

3 Exam-style questions

3.1 The r.m.s. value of a sinusoidal a.c. of peak I_0 is: [1]

- **A** I_0
- **B** $I_0/2$
- **C** $I_0/\sqrt{2}$
- **D** $2I_0$

3.2 A sinusoidal supply has a peak voltage of 340 V. Calculate (a) the r.m.s. voltage, (b) the mean power delivered to a 100 Ω resistor. [4]

3.3 Explain why the **mean power** in a resistor is **half** the peak power for a sinusoidal a.c. [2]

3.4 An a.c. has a period of 0.020 s. Find the **angular frequency** ω. [2]

3.5 A mains voltage is $V = 340\sin(100\pi t)$, with V in volts and t in seconds.

(a) Show that the **period** is 0.020 s. [2]

(b) Calculate the **r.m.s. voltage** and the **mean power** delivered to a 60 Ω heater. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- **peak value** — the maximum value in a cycle
- **period** — the time for one complete cycle

2.2

- $I_{\text{r.m.s.}} = I_0/\sqrt{2}$

$$I_{\text{r.m.s.}} = \frac{5.0\ \text{A}}{\sqrt{2}} = \mathbf{3.5}\ \text{A}$$

2.3

- $V_{\text{r.m.s.}} = V_0/\sqrt{2}$

2.4

- $T = 1/f = 1/(50\ \text{Hz}) = \mathbf{0.020}\ \text{s}$

2.5

- peak $V_0 = 18$ V
- $\omega = 40\pi$ rad s^{-1} (≈ 126 rad s^{-1})

3.1 C

- $I_{\text{r.m.s.}} = I_0/\sqrt{2}$

3.2

(a) $V_{\text{r.m.s.}} = V_0/\sqrt{2}$

$$V_{\text{r.m.s.}} = \frac{340\ \text{V}}{\sqrt{2}} = \mathbf{240}\ \text{V}$$

(b) $P = V_{\text{r.m.s.}}^2/R$

$$P = \frac{(240\ \text{V})^2}{100\ \Omega} = \mathbf{576}\ \text{W}$$

3.3

- the power $P = I^2R$ varies as $\sin^2$, whose **average over a cycle is** $\frac{1}{2}$
- so the mean power is half the peak power I_0^2R

3.4

- $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.020\ \text{s}} = \mathbf{3.1 \times 10^2}\ \text{rad s}^{-1}$$

3.5

(a) $\omega = 100\pi$ rad s^{-1}

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi\ \text{rad s}^{-1}} = \mathbf{0.020}\ \text{s}$$

(b) $V_{\text{r.m.s.}} = 340\ \text{V}/\sqrt{2} = 240$ V

$$\langle P\rangle = \frac{V_{\text{r.m.s.}}^2}{R} = \frac{(240\ \text{V})^2}{60\ \Omega} = \mathbf{9.6 \times 10^2}\ \text{W}$$