

20.5.1 Induced e.m.f. from a graph: gradients, the rotating coil and Lenz in action

Name: _____ Class: _____ Date: _____

Total: 31 marks

KEY WORDS magnetic flux 磁通量 • flux linkage 磁链 • faraday's law 法拉第定律

• lenz's law 楞次定律 • magnetic field 磁场

Objective

Sheet 20.5 gave you the definitions and the standard $E = -\frac{\Delta(N\Phi)}{\Delta t}$ calculation. This sheet is the other half of what 20.5 is examined on, and it is almost all **graphs**: reading an e.m.f. off the **gradient** of a flux-linkage graph and sketching one from the other, the two-pulse trace a magnet makes falling through a coil, the **sinusoidal** e.m.f. of a **rotating coil**, and the qualitative Lenz's-law questions – the magnet in a copper tube, the jumping ring, **eddy-current damping** 涡流阻尼. This sheet covers syllabus 20.5.

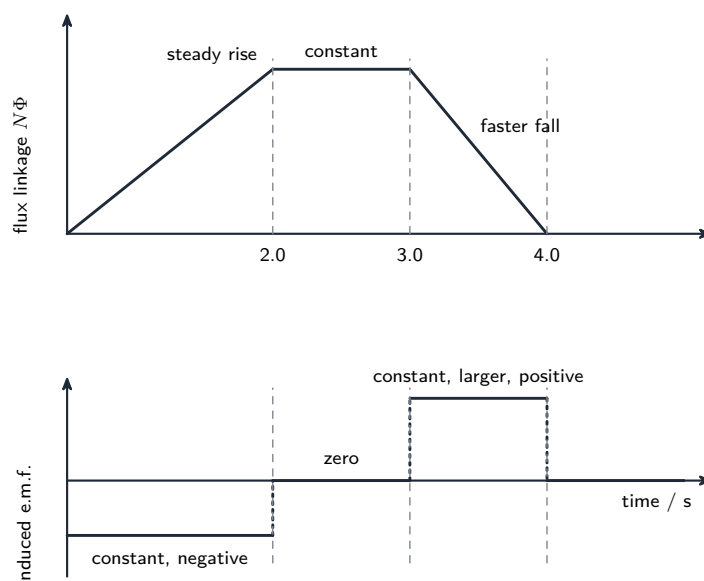
You must be able to:

- define **magnetic flux** 磁通量 and recall and use $\Phi = BA$, and use **magnetic flux linkage** 磁链 $N\Phi$
- understand and explain experiments showing that a **changing** magnetic flux induces an e.m.f., that the induced e.m.f. opposes the change producing it, and what affects its magnitude
- recall and use **Faraday's law** and **Lenz's law**, including reading and sketching graphs of flux linkage and of induced e.m.f. against time

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

- Faraday's law is a statement about a gradient



Faraday's law is a statement about a **gradient**: $E = -\frac{d(N\Phi)}{dt}$. A straight section of the upper graph gives a *constant* e.m.f.; a flat section gives *zero*; a steeper section gives a *bigger* one. The minus sign is Lenz's law, and it flips the sign of the e.m.f. when the flux starts to fall.

$$E = -\frac{d(N\Phi)}{dt}$$

Read the lower graph straight off the **gradient** of the upper one:

the flux-linkage graph	the induced e.m.f.
a straight sloping section	constant , and not zero
a flat section	zero – the flux is not changing, so no e.m.f. at all
a steeper section	larger in magnitude
slope changes sign	the e.m.f. changes sign

- **Faraday's and Lenz's laws of electromagnetic induction** are the two halves of this one equation: Faraday's law fixes the **size** of the e.m.f. (proportional to the rate of change of flux linkage), and Lenz's law fixes its **direction** (it opposes the change producing it).
- The minus sign **is** Lenz's law. It is not decoration: it is what makes the e.m.f. reverse when the flux starts to fall.
- A flat section is where marks are most often lost. A large, constant flux induces **nothing**.

Worked example. A coil of $N = 400$ turns and area $2.5 \times 10^{-3} \text{ m}^2$ sits with its plane perpendicular to a field that falls steadily from 0.60 T to zero in 0.15 s . Find the magnitude of the induced e.m.f.

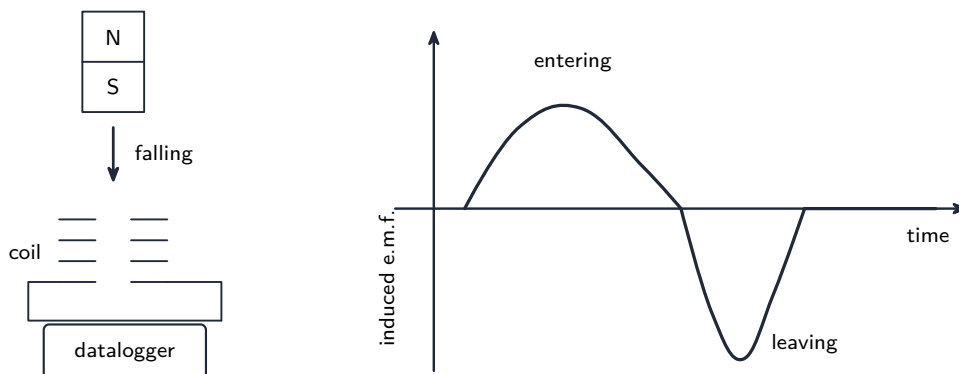
- the flux linkage falls from NBA to zero

$$N\Phi = 400 \times 0.60 \text{ T} \times 2.5 \times 10^{-3} \text{ m}^2 = 0.60 \text{ Wb}$$

- divide the change by the time

$$|E| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{0.60 \text{ Wb}}{0.15 \text{ s}} = 4.0 \text{ V}$$

■ A magnet dropped through a coil

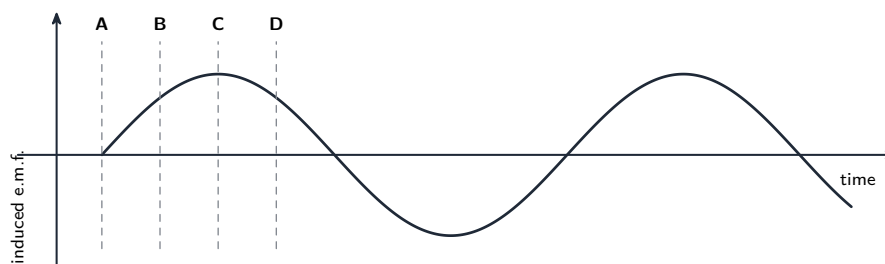
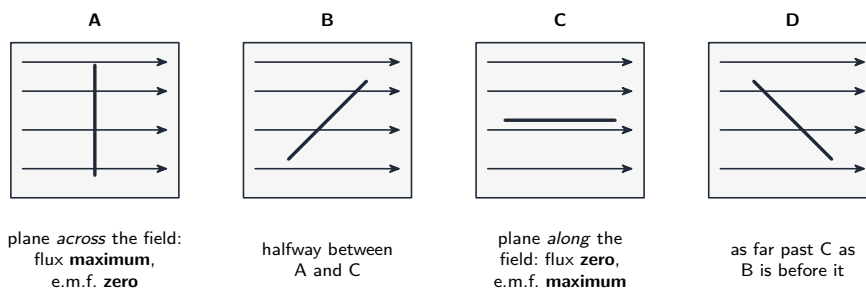


Three features, three marks. The two pulses have **opposite signs**, because the flux first rises and then falls. The second is **taller and narrower**, because the magnet is moving faster, so the same flux change happens in less time. The two **areas are equal**, because each is the same total change of flux linkage.

Three features, three separate marks:

1. **Two pulses of opposite sign.** As the magnet enters, the flux linkage **rises**; as it leaves, the flux linkage **falls**. Opposite gradients give opposite e.m.f.s.
 2. **The second pulse is taller and narrower.** The magnet has **accelerated under gravity**, so it is moving faster when it leaves: the same change of flux happens in **less time**, so $\frac{d(N\Phi)}{dt}$ is larger and lasts for less time.
 3. **The two areas are equal.** The area under an e.m.f.-time graph is $\int E dt = \Delta(N\Phi)$, and the flux change entering equals the flux change leaving.
- Between the pulses the e.m.f. passes through **zero**, at the moment the magnet is symmetrically inside the coil and the flux linkage is momentarily stationary.

■ A coil rotating in a uniform field



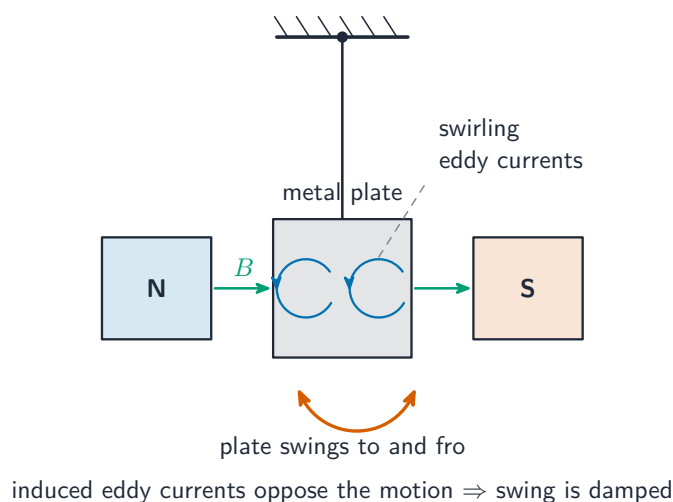
At **A** the flux through the coil is greatest, and the e.m.f. is **zero**: the flux is momentarily not changing. At **C** the flux is zero and the e.m.f. is **greatest**: the flux is changing fastest. The e.m.f. follows the *rate of change* of flux, so the two graphs are a quarter of a cycle apart.

With the coil turning at a steady rate, $N\Phi = NBA \cos(2\pi ft)$, so

$$E = -\frac{d(N\Phi)}{dt} = 2\pi f NBA \sin(2\pi ft), \quad \text{so} \quad E_0 = 2\pi f NBA$$

- The e.m.f. is **zero** when the flux linkage is **greatest** (plane of the coil **across** the field), and **greatest** when the flux linkage is **zero** (plane of the coil **along** the field). The two graphs are a **quarter of a cycle** apart.
- Double the rotation rate and the peak e.m.f. **doubles** ($E_0 \propto f$) while the period **halves**. Both changes come from the same f .
- This is the a.c. generator, and it is the commonest place the “e.m.f. is biggest when the flux is biggest” error appears.

■ Lenz’s law when nothing is being calculated



Every qualitative question is the same four steps. Write them in this order:

1. The motion **changes the flux** linking the conductor.
2. So an e.m.f. is induced, and because the conductor is a complete circuit, a **current** flows.
3. By **Lenz's law** the current is in the direction that **opposes the change** producing it.
4. So there is a **force opposing the motion**, and the moving object slows down.

what you are shown	what to say
a magnet falls slowly down a copper tube	the changing flux induces eddy currents 涡电流 in the tube wall; by Lenz's law they oppose the magnet's motion, so there is an upward force on the magnet and it falls at nearly constant, low speed
a plastic tube instead	plastic is an insulator : an e.m.f. is still induced but no current can flow, so there is no opposing force and the magnet falls freely
an aluminium ring jumps off an electromagnet switched on	the rising flux induces a current in the ring, opposing the increase, so the ring's field opposes the magnet's and it is repelled
a slotted disc is not damped	the slots break the path of the eddy currents, so almost no current flows

- Finish with **energy**: the work done against the opposing force is transferred to **electrical energy** and then to **thermal energy** in the conductor's resistance. If the induced current helped the motion instead, energy would be created from nothing.

2 Practice

Now use the methods above.

2.1 A coil of 150 turns and cross-sectional area $4.0 \times 10^{-3} \text{ m}^2$ has its plane perpendicular to a uniform field of flux density 0.25 T. Calculate the flux linkage. [2]

2.2 The flux density in **2.1** then falls steadily to zero in 0.20 s. Calculate the magnitude of the induced e.m.f. [2]

2.3 The flux linkage through a coil rises steadily for 2.0 s, stays constant for 1.0 s, and then falls to zero in 1.0 s. Sketch a graph of the induced e.m.f. against time for these 4.0 s. [3]

2.4 A magnet is dropped through a coil connected to a datalogger. Explain why the trace shows two pulses of **opposite sign**. [2]

2.5 Explain why the second pulse is **taller and narrower** than the first. [2]

2.6 A coil rotates at a steady rate in a uniform magnetic field. State the position of the coil at which the induced e.m.f. is **zero**, and explain why. [2]

3 Exam-style questions

3.1 A coil rotates at a steady rate in a uniform magnetic field. At the instant when the **plane of the coil is perpendicular to the field**, the flux linkage and the induced e.m.f. are [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** flux linkage maximum, e.m.f. maximum
B flux linkage maximum, e.m.f. zero
C flux linkage zero, e.m.f. maximum
D flux linkage zero, e.m.f. zero

3.2 A strong bar magnet is dropped down a long vertical **copper** tube. It takes several seconds to fall through. The same magnet dropped down an identical **plastic** tube falls through in a fraction of a second.

(a) Explain, in terms of flux, induced current and force, why the magnet falls slowly down the copper tube. [4]

(b) Explain why the magnet falls freely down the plastic tube. [2]

3.3 A flat coil of 250 turns and area $1.8 \times 10^{-3} \text{ m}^2$ rotates at 50 revolutions per second about an axis perpendicular to a uniform magnetic field of flux density 85 mT.

(a) Calculate the maximum flux linkage through the coil. [2]

(b) State the orientation of the coil when the flux linkage is a maximum, and state the induced e.m.f. at that instant. [2]

(c) On the same time axis, sketch the flux linkage and the induced e.m.f. against time, for one complete rotation. [3]

(d) The peak e.m.f. is given by $E_0 = 2\pi f NBA$. Calculate it. [2]

(e) The coil is now rotated at 100 revolutions per second. State the effect on the peak e.m.f. and on the period, and explain both. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the plane is perpendicular to the field, so the full area links the flux

$$N\Phi = NBA = 150 \times 0.25 \text{ T} \times 4.0 \times 10^{-3} \text{ m}^2 = \mathbf{0.15 \text{ Wb}}$$

2.2

- divide the change in flux linkage by the time

$$|E| = \frac{\Delta(N\Phi)}{\Delta t} = \frac{0.15 \text{ Wb}}{0.20 \text{ s}} = \mathbf{0.75 \text{ V}}$$

2.3

- from 0 to 2.0 s: a **constant** e.m.f., of one sign
- from 2.0 to 3.0 s: **zero**
- from 3.0 to 4.0 s: **constant**, of the **opposite** sign and **twice** the earlier magnitude, because the same flux change happens in half the time

2.4

- as the magnet enters, the flux linkage through the coil **increases**; as it leaves, the flux linkage **decreases**
- the gradient $\frac{d(N\Phi)}{dt}$ therefore has **opposite signs**, so the induced e.m.f. does too

2.5

- the magnet **accelerates** under gravity, so it is moving faster as it leaves than as it entered
- the same change of flux linkage therefore happens in a **shorter time**, giving a **larger** e.m.f. that lasts for **less time** – the two areas remain equal

2.6

- when the **plane of the coil is perpendicular to the field** – that is, when the flux linkage is a **maximum**
- at that instant the flux linkage is momentarily **not changing**, so $\frac{d(N\Phi)}{dt} = 0$ and the induced e.m.f. is zero

3.1 B

- with the plane of the coil perpendicular to the field, the full area links the flux, so the flux linkage is at its **maximum**
- a maximum is a turning point, so the flux linkage is momentarily constant and the e.m.f. is **zero**. **A** is the standard error: the e.m.f. follows the *rate of change*, not the flux

3.2

(a)

- as the magnet falls, the magnetic flux linking each part of the copper tube **changes**
- an e.m.f. is induced, and because copper is a conductor a current – an **eddy current** – flows in the tube wall
- by **Lenz's law** the current is in the direction that **opposes the change** producing it
- so the tube exerts an **upward force** on the falling magnet, which nearly balances its weight, and the magnet falls slowly at almost constant speed

(b)

- an e.m.f. is still induced in the plastic, because the changing flux does not care what the material is
- but plastic is an **insulator**, so no current flows, there is no opposing force, and the magnet falls freely under gravity

3.3

(a)

- the maximum flux linkage is NBA , with the plane of the coil perpendicular to the field

$$N\Phi_{\max} = 250 \times 85 \times 10^{-3} \text{ T} \times 1.8 \times 10^{-3} \text{ m}^2 = \mathbf{3.8 \times 10^{-2} \text{ Wb}}$$

(b)

- the **plane of the coil is perpendicular to the field** (the normal to the coil is along the field)
- at that instant the induced e.m.f. is **zero**

(c)

- flux linkage: a **cosine** curve, starting at its maximum $3.8 \times 10^{-2} \text{ Wb}$, through zero, to $-3.8 \times 10^{-2} \text{ Wb}$ and back
- e.m.f.: a **sine** curve of the same period
- the e.m.f. curve is a **quarter of a cycle** ahead: its zeros are at the peaks of the flux-linkage curve, and its peaks are at the zeros

(d)

- substitute into $E_0 = 2\pi f NBA$, using the flux linkage from part (a)

$$E_0 = 2\pi \times 50 \text{ Hz} \times 3.8 \times 10^{-2} \text{ Wb} = \mathbf{12 \text{ V}}$$

(e)

- the peak e.m.f. **doubles**, to about 24 V, because $E_0 = 2\pi f NBA$ is proportional to f
- the period **halves**, to $1.0 \times 10^{-2} \text{ s}$, because $T = 1/f$; both changes follow from the same doubling of f