A-LEVEL PHYSICS • EXERCISE SHEET

20.3.1 The Hall effect, the Hall probe and velocity selection

Name: ____________ Class: ________ Date: __________ **Total: 30 marks**

KEY WORDS hall voltage 霍尔电压 • electric field 电场 • hall probe 霍尔探头 • magnetic flux density 磁通量密度 • velocity 速度

Objective

Sheet 20.3 covered $F = BQv\sin\theta$ and the circular path. This sheet is the rest of 20.3, and it is the part with the **derivations**: where the **Hall voltage** 霍尔电压 comes from and how $V_{\mathrm{H}} = \dfrac{BI}{ntq}$ is obtained, how a **Hall probe** 霍尔探头 measures a **magnetic flux density** 磁通量密度, and how crossed electric and magnetic fields **select one speed** out of a beam. This sheet covers syllabus 20.3.

You must be able to:

- determine the direction of the force on a charge moving in a magnetic field, and recall and use $F = BQv\sin\theta$
- understand the origin of the Hall voltage, and **derive** and use $V_{\mathrm{H}} = \dfrac{BI}{ntq}$, where t is the thickness
- understand the use of a Hall probe to measure magnetic flux density
- describe the motion of a charged particle in a uniform magnetic field perpendicular to its motion
- explain how electric and magnetic fields are used in **velocity selection** 速度选择

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

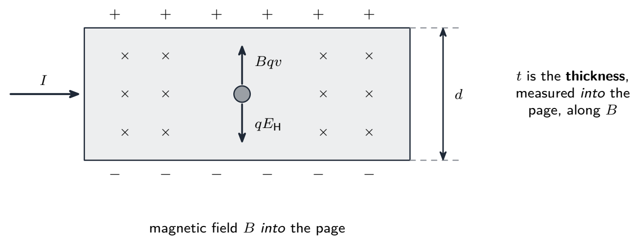
What you already have, in two lines

- $F = BQv\sin\theta$, with θ measured between the **velocity and the field**. The force is **zero** when the charge moves **along** the field ($\theta = 0$), and **maximum** when it moves at right angles.
- Perpendicular to a uniform field, the force is always at right angles to the velocity, so it is a **centripetal** force and the path is a **circle**:

$$BQv = \frac{mv^2}{r} \quad\Longrightarrow\quad r = \frac{mv}{BQ}$$

- The speed never changes, because a force at right angles to the motion does **no work**.

Where the Hall voltage comes from, and the derivation



The magnetic force pushes the carriers to one face until the electric field they build pushes back just as hard: $qE_{\mathrm{H}} = Bqv$, so $E_{\mathrm{H}} = Bv$ and $V_{\mathrm{H}} = E_{\mathrm{H}}d = Bvd$. Substituting the drift speed from $I = nAvq$ with $A = td$ gives $V_{\mathrm{H}} = \dfrac{BI}{ntq}$.

The derivation is four steps, and each one is a marking point:

1. A carrier drifting with speed v through the field feels a magnetic force Bqv, which pushes it to **one face** of the slab.
2. Charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** E_{H} appears **across** the slab.
3. Charge stops building up when the **electric force balances the magnetic force**:
$$qE_{\mathrm{H}} = Bqv \quad\Longrightarrow\quad E_{\mathrm{H}} = Bv$$
and since the field is uniform across the width d, the **Hall voltage** is
$$V_{\mathrm{H}} = E_{\mathrm{H}}\,d = Bvd$$
4. Eliminate the drift speed using $I = nAvq$ with $A = td$:
$$v = \frac{I}{ntdq} \quad\Longrightarrow\quad V_{\mathrm{H}} = B\left(\frac{I}{ntdq}\right)d = \frac{BI}{ntq}$$

- n is the **number density** of charge carriers and t is the **thickness measured along the field**, not the width.
- Notice the d **cancels**: the Hall voltage does not depend on the width of the slab.

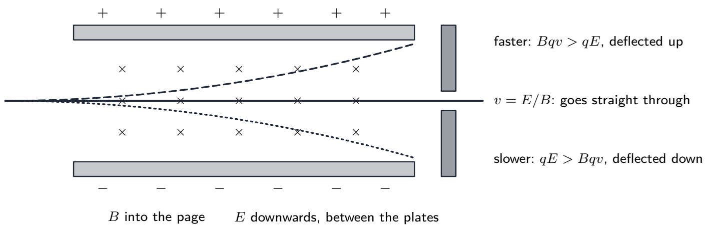
Why a Hall probe is made of semiconductor

$$V_{\mathrm{H}} = \frac{BI}{ntq}$$

- With I, t and q fixed, $V_{\mathrm{H}} \propto B$: measure the voltage and you have measured the **flux density**. That is the whole instrument.

- A **semiconductor** 半导体 has a number density n perhaps 10^7 times smaller than a metal's, and $V_{\mathrm{H}} \propto \frac{1}{n}$, so the voltage is large enough to measure. A metal probe would give microvolts.
- Making the slab **thin** helps for the same reason.
- In use: the probe must be held with its **face perpendicular to the field**, because it responds to the component of B through it, and the current must be kept **constant**.

■ Velocity selection



The electric force qE and the magnetic force Bqv act in *opposite* directions, and they balance only when $qE = Bqv$, that is when $v = \frac{E}{B}$. Every other speed is deflected and stopped by the slit, whatever the particle's charge or mass.

Put an electric field and a magnetic field at right angles to each other and to the beam, so that the two forces on a charge are in **opposite** directions:

$$qE = Bqv \quad \Longrightarrow \quad v = \frac{E}{B}$$

- Only a charge with **exactly** this speed passes through undeflected and reaches the slit.
- **Faster:** the magnetic force Bqv grows with v while the electric force does not, so $Bqv > qE$ and the charge is deflected towards the magnetic force.
- **Slower:** $qE > Bqv$, and it is deflected the other way.
- The charge q **cancels**, so the selected speed does not depend on the charge or the mass – the same selector works on any ion.

2 Practice

Now use the methods above.

2.1 Write the equation for the force on a charge moving at an angle to a magnetic field, and state the condition for the force to be zero. [2]

2.2 Explain why a charged particle entering a uniform magnetic field at right angles to it follows a circular path at constant speed. [3]

2.3 Explain how a Hall voltage arises across a slab carrying a current in a magnetic field. [3]

2.4 Explain why a Hall probe is made from a semiconductor rather than a metal. [2]

2.5 State the condition for a charged particle to pass undeflected through a velocity selector, and give the expression for the selected speed. [2]

3 Exam-style questions

3.1 A Hall probe is to give the largest possible Hall voltage for a given flux density. The slab should have [1]

A a large number density of charge carriers and a large thickness.
B a large number density of charge carriers and a small thickness.
C a small number density of charge carriers and a large thickness.
D a small number density of charge carriers and a small thickness.

3.2 A rectangular slab of width d and thickness t carries a current I. A uniform magnetic field of flux density B acts perpendicular to the face of the slab, and the charge carriers each have charge q and number density n.

(a) Derive the expression $V_{\mathrm{H}} = \dfrac{BI}{ntq}$ for the Hall voltage across the slab. [4]

(b) A Hall probe uses a slab of thickness 0.10 mm in which $n = 2.0 \times 10^{22}$ m^{-3} and $q = 1.6 \times 10^{-19}$ C. The current through it is 15 mA. Calculate the Hall voltage when the probe is placed in a field of flux density 0.25 T. [2]

3.3 Positive ions, each of charge $+1.6 \times 10^{-19}$ C, pass through a velocity selector in which the electric field strength is 3.0×10^{4} V m^{-1} and the magnetic flux density is 0.12 T. They then enter a second region of uniform magnetic field of flux density 0.35 T, perpendicular to their motion.

(a) Calculate the speed of the ions that pass through the selector undeflected. [2]

(b) Explain why ions travelling faster than this do not pass through. [2]

(c) In the second region an ion follows a circular path of radius 0.086 m. Calculate its mass. [3]

(d) State and explain the effect on the radius of the path of using an ion of the same charge and speed but **twice** the mass. [2]

(e) Explain why the magnetic field in the second region does no work on the ion. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $F = BQv\sin\theta$, where θ is the angle between the velocity and the magnetic field
- the force is zero when $\theta = 0$, that is when the charge moves **along** the field

2.2

- the magnetic force is always at **right angles to the velocity**, so it changes the direction of motion but not its magnitude
- a force of constant magnitude always perpendicular to the velocity is a **centripetal** force, so the path is a circle
- since the force is perpendicular to the motion it does **no work**, so the kinetic energy and therefore the speed are constant

2.3

- the drifting charge carriers experience a magnetic force Bqv that pushes them towards **one face** of the slab
- charge builds up on that face, leaving the opposite face oppositely charged, so a **transverse electric field** appears across the slab
- the build-up stops when the **electric force on a carrier balances the magnetic force**, and the p.d. across the slab at that point is the Hall voltage

2.4

- $V_H = \dfrac{BI}{ntq}$, so the Hall voltage is **inversely proportional to the number density** n
- a semiconductor has a far smaller n than a metal, so the voltage is large enough to be measured

2.5

- the **electric force and the magnetic force must be equal and opposite**, so $qE = Bqv$

$$v = \frac{E}{B}$$

3.1 D

- $V_H = \dfrac{BI}{ntq}$, so a **small** n and a **small** t both make V_H larger
- **A** and **B** would give a tiny voltage, which is why a metal is useless as a Hall probe

3.2

(a)

- a carrier drifting at speed v feels a magnetic force Bqv pushing it to one face, so charge collects there and a transverse electric field E_H builds up

- equilibrium is reached when the electric force balances the magnetic force

$$qE_H = Bqv \implies E_H = Bv, \qquad V_H = E_H d = Bvd$$

- eliminate v with $I = nAvq$ and $A = td$

$$v = \frac{I}{ntdq} \implies V_H = B\frac{I}{ntdq}d = \frac{BI}{ntq}$$

(b)

- substitute, with the thickness in metres

$$V_H = \frac{0.25\ \text{T} \times 0.015\ \text{A}}{2.0 \times 10^{22}\ \text{m}^{-3} \times 1.0 \times 10^{-4}\ \text{m} \times 1.6 \times 10^{-19}\ \text{C}} = \mathbf{1.2 \times 10^{-2}}\ \text{V}$$

3.3

(a)

- the two forces balance, so $qE = Bqv$ and the charge cancels

$$v = \frac{E}{B} = \frac{3.0 \times 10^4\ \text{V m}^{-1}}{0.12\ \text{T}} = \mathbf{2.5 \times 10^5}\ \text{m s}^{-1}$$

(b)

- the magnetic force Bqv **increases with speed**, while the electric force qE does not
- so for a faster ion $Bqv > qE$, the forces no longer balance, and the ion is deflected out of the beam and stopped by the slit

(c)

- the magnetic force provides the centripetal force

$$Bqv = \frac{mv^2}{r} \implies m = \frac{rBq}{v}$$

$$m = \frac{0.086\ \text{m} \times 0.35\ \text{T} \times 1.6 \times 10^{-19}\ \text{C}}{2.5 \times 10^5\ \text{m s}^{-1}} = \mathbf{1.9 \times 10^{-26}}\ \text{kg}$$

(d)

- the radius **doubles**
- $r = \dfrac{mv}{Bq}$, and with v, B and q unchanged the radius is proportional to the mass

(e)

- the magnetic force is always **perpendicular to the velocity**, so it has no component along the direction of motion
- work done is $Fs\cos\theta$ with $\theta = 90°$, so it is zero; the kinetic energy and the speed are unchanged and only the direction alters