A-LEVEL PHYSICS • EXERCISE SHEET

2.1 Equations of motion

Name: ____________ Class: ________ Date: __________ **Total: 49 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • air resistance 空气阻力 • speed 速率 • distance 距离

Objective

Build the skills to answer exam questions on **uniformly accelerated motion 匀加速运动** — choosing and using the SUVAT equations, reading motion graphs, and solving free-fall and projectile problems.

You must be able to:

- find velocity from a displacement–time gradient, and acceleration + displacement from a velocity–time graph
- choose and use the four SUVAT equations for straight-line motion
- solve free-fall problems ($g = 9.81$ m s^{-2}, no air resistance)
- treat the horizontal and vertical motion of a projectile independently
- **describe** the experiment that measures the acceleration of free fall
- **sketch** a motion graph with both axes drawn and labelled

1 Worked examples

Study these first. Each one shows the **method 方法** for a question type used later — examiners give marks for showing the working, not only the final number.

Choosing a SUVAT equation

The five symbols are u (start velocity), v (final velocity), a (acceleration), s (displacement) and t (time). Each equation uses **four** of them and leaves one out:

equation	leaves out
$v = u + at$	s
$s = \frac{1}{2}(u+v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t

Method: list what you know and what you want, then pick the row that leaves out the symbol you don't have.

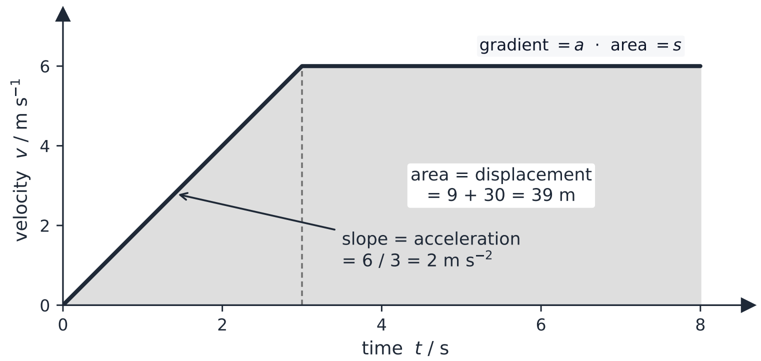
Example. A cyclist speeds up uniformly from 4.0 to 13 m s^{-1} in 6.0 s: so $u = 4.0$,

$v = 13$, $t = 6.0$; find a, then s.

- For a (no s): $v = u + at \Rightarrow a = \dfrac{v-u}{t} = \dfrac{13-4.0}{6.0} = 1.5$ m s^{-2}.
- For s (no a): $s = \frac{1}{2}(u+v)t = \frac{1}{2}(4.0+13)(6.0) = 51$ m.

Reading a velocity–time graph

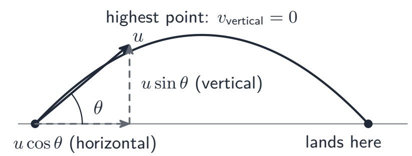
On a velocity–time graph the **gradient 斜率** (steepness) is the acceleration, and the **area** under the line is the displacement.



(For an *acceleration*–time graph, the area under the line is instead the *change in velocity*.)

A projectile

Horizontal and vertical motion are **independent** — they share only the time t, so solve each as its own SUVAT problem. For a launch at angle θ, first **resolve** the speed: horizontal $u\cos\theta$ (stays constant) and vertical $u\sin\theta$ (changes).



Worked case — a ball thrown **horizontally** at 12 m s^{-1} from a cliff 25 m high ($g = 9.81$ m s^{-2}):

- Vertical (start 0): $25 = \frac{1}{2}g\,t^2 \Rightarrow t = \sqrt{2\times 25/9.81} = 2.3$ s.
- Horizontal: $x = 12 \times 2.3 = 27$ m.

Measuring the acceleration of free fall

The syllabus asks you to **describe** this experiment, so learn it as five steps:

1. A steel ball is held by an **electromagnet** above a **trapdoor** switch, a measured height h above it.
2. Switching the magnet off starts an electronic **timer** and releases the ball at the same instant.
3. The ball opens the trapdoor on landing, which **stops** the timer, giving the fall time t.
4. The ball falls from rest, so $h = \frac{1}{2}gt^2$. Repeat for **several heights**, plot h against t^2, and the **gradient** is $\frac{1}{2}g$.
5. Measure h from the **bottom** of the ball to the trapdoor, and repeat each timing — the graph, not a single pair of readings, is what removes the random error.

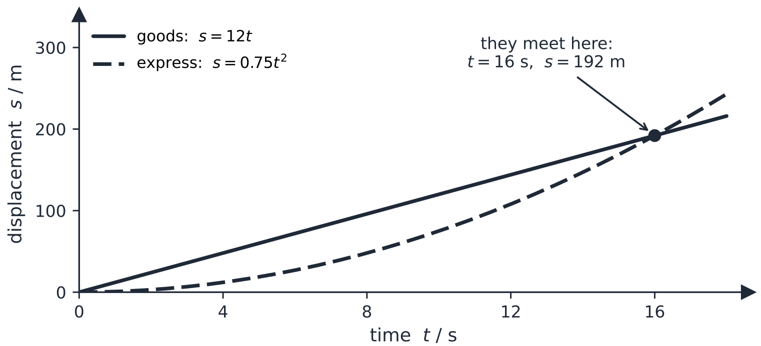
A steel ball is used because **air resistance** on it is negligible; the graph is a straight line **through the origin** only if there is no systematic timing delay.

Two objects meeting

Write a displacement equation for each object using the same start time and positive direction, then set the displacements equal.

A goods train moves at a constant 12 m s^{-1}. As it passes a signal, an express train starts from rest there with acceleration 1.5 m s^{-2}.

- Goods: $s = 12t$. Express: $s = \frac{1}{2}(1.5)t^2 = 0.75t^2$.
- Level when $12t = 0.75t^2 \Rightarrow t = \dfrac{12}{0.75} = 16$ s ($s = 12 \times 16 = 192$ m).



2 Practice

Now apply the methods above.

2.1 For each quantity, write **scalar** 标量 or **vector** 矢量, and state its SI unit: (a) velocity (b) acceleration. [2]

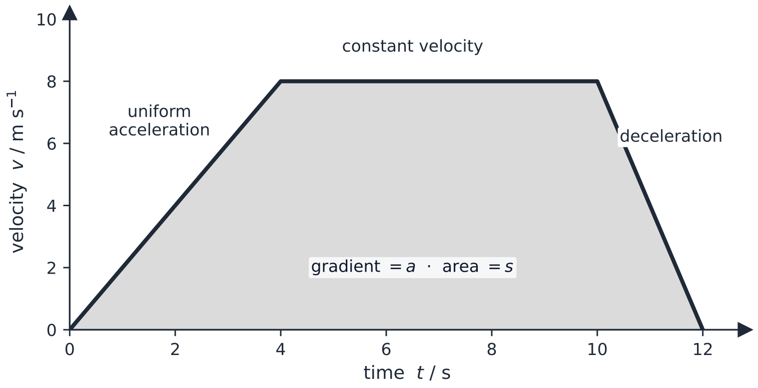
quantity	scalar or vector?	SI unit
velocity		
acceleration		

2.2 A car travelling at 24 m s^{-1} brakes uniformly at 3.0 m s^{-2} until it stops. Find the braking distance. *(Hint: which SUVAT equation links u, v, a and s?)* [3]

2.3 The displacement–time graph of a cyclist is a straight line passing through (2.0 s, 12 m) and (8.0 s, 42 m). **Determine** her velocity, and state how you know her acceleration is zero. [3]

2.4 **Describe** an experiment to determine the acceleration of free fall using a falling object. State the measurements you would take, how you would use them, and one way to reduce the uncertainty. [4]

2.5 The velocity–time graph shows a tram's journey.



Velocity–time graph for the tram's three-phase journey

(a) Find the acceleration during the first 4 s. [1]

(b) Find the total displacement of the tram. *(Hint: split the area into a triangle, a rectangle and a triangle.)* [3]

3 Exam-style questions

3.1 A stone is thrown vertically upwards at 15 m s^{-1}. Take $g = 9.81$ m s^{-2} and ignore air resistance. Its height above the launch point after 2.0 s is closest to: [1]

- **A** 10 m
- **B** 20 m
- **C** 30 m
- **D** 50 m

3.2 The area under an **acceleration–time graph** gives which quantity? [1]

- **A** displacement
- **B** change in velocity
- **C** distance
- **D** force

3.3 A train starts from rest and accelerates uniformly at 0.50 m s^{-2} for 40 s.

(a) Calculate the speed it reaches. [2]

(b) Calculate the distance travelled during these 40 s. [2]

(c) State one assumption you have made in your calculation. [1]

3.4 A ball is kicked from level ground at 20 m s^{-1}, at 30° above the horizontal. Take $g = 9.81$ m s^{-2} and ignore air resistance.

(a) Show that the vertical **component** 分量 of the initial velocity is about 10 m s^{-1}. [1]

(b) Calculate the time the ball is in the air before it lands. [2]

(c) Calculate the horizontal distance it travels (its range). [2]

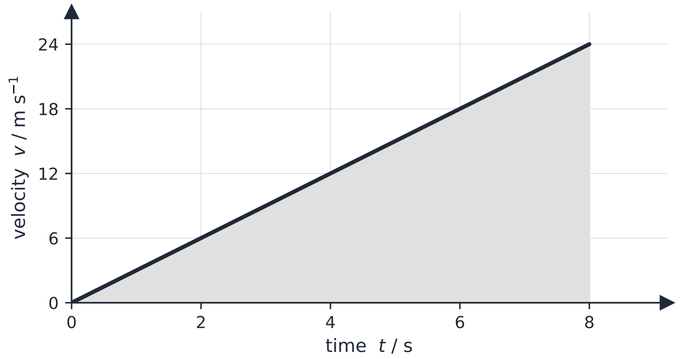
3.5 A car passes a stationary motorcyclist at a constant 18 m s^{-1}. At that instant the motorcyclist sets off from rest with a uniform acceleration of 3.0 m s^{-2}.

(a) Calculate the time the motorcyclist takes to catch up with the car. [3]

(b) Calculate how far each has travelled at that moment. [1]

(c) Calculate the motorcyclist's speed when she catches the car. [1]

3.6 A drone of mass 1.8 kg takes off from the ground and rises vertically. The graph shows its velocity for the first 8.0 s of the flight.



(a) **Define** acceleration. [1]

(b) **Determine** the acceleration of the drone. [2]

(c) **Show that** the drone rises 96 m in these 8.0 s. [2]

(d) Calculate the gain in **gravitational potential energy** of the drone. [2]

(e) Calculate the gain in **kinetic energy** of the drone. [2]

(f) **Determine** the average useful power output of the drone's motors during these 8.0 s. [2]

(g) In the space below, **sketch** the variation with time of the **displacement** of the drone over the same 8.0 s. **Draw** and **label** both axes. [3]

(h) A second drone of mass 2.7 kg takes off and follows exactly the same velocity–time graph. **State and explain** whether the average useful power output of its motors is less than, the same as, or greater than your answer to **(f)**. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- velocity is a **vector**, unit m s^{-1}
- acceleration is a **vector**, unit m s^{-2}

2.2

- known: $u = 24$ m s^{-1}, $v = 0$, $a = -3.0$ m s^{-2}; want s (no t)
- use $v^2 = u^2 + 2as$

$$0 = (24 \text{ m s}^{-1})^2 + 2(-3.0 \text{ m s}^{-2})s \Rightarrow s = \frac{576 \text{ m}^2\text{ s}^{-2}}{6.0 \text{ m s}^{-2}} = \mathbf{96} \text{ m}$$

2.3

- velocity is the **gradient** of a displacement–time graph

$$v = \frac{42 \text{ m} - 12 \text{ m}}{8.0 \text{ s} - 2.0 \text{ s}} = \frac{30 \text{ m}}{6.0 \text{ s}} = \mathbf{5.0} \text{ m s}^{\mathbf{-1}}$$

- the graph is a **straight** line, so v never changes; acceleration is the rate of change of velocity, so $a = 0$

2.4

- hold a steel ball with an electromagnet a measured height h above a trapdoor
- switching the magnet off starts an electronic timer and releases the ball; the ball opens the trapdoor and stops the timer, giving t
- use $h = \frac{1}{2}gt^2$, or plot h against t^2 and take $g = 2\times$ gradient
- reduce uncertainty: repeat each timing, and use several heights so a graph averages the readings

2.5

(a) acceleration = gradient of the v–t graph

$$a = \frac{8.0 \text{ m s}^{-1} - 0}{4.0 \text{ s}} = \mathbf{2.0} \text{ m s}^{\mathbf{-2}}$$

(b) displacement = area under the line; split into triangle + rectangle + triangle

- 0–4 s: $\frac{1}{2} \times 4.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 16$ m
- 4–10 s: $6.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 48$ m
- 10–12 s: $\frac{1}{2} \times 2.0 \text{ s} \times 8.0 \text{ m s}^{-1} = 8$ m
- total = 16 m + 48 m + 8 m = **72** m

3.1 A

- take up as positive: $u = 15$ m s^{-1}, $a = -9.81$ m s^{-2}, $t = 2.0$ s

$s = ut + \frac{1}{2}at^2 = (15 \text{ m s}^{-1})(2.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m s}^{-2})(2.0 \text{ s})^2 = 30 \text{ m} - 19.6 \text{ m} = 10.4 \text{ m}$

- closest to **10** m
- **B** forgot the $\frac{1}{2}at^2$ term; **C** is ut alone

3.2 B

- the area under an **acceleration–time** graph is the **change in velocity**
- **A** / **C** are the area under a **velocity–time** graph

3.3

(a) $u = 0$, $a = 0.50$ m s^{-2}, $t = 40$ s

$$v = u + at = 0 + (0.50 \text{ m s}^{-2})(40 \text{ s}) = \mathbf{20} \text{ m s}^{-1}$$

(b) no v needed: $s = ut + \frac{1}{2}at^2$

$$s = \tfrac{1}{2}(0.50 \text{ m s}^{-2})(40 \text{ s})^2 = \mathbf{400} \text{ m}$$

(c) any one: the acceleration stays constant / the track is straight / no resistive forces

3.4

(a) vertical component $= u\sin\theta$

- $u_V = 20$ m s$^{-1} \times \sin 30° = 20 \times 0.50 = \mathbf{10}$ m s^{-1}

(b) time to the top: $v_V = 0 = u_V - gt$

$$t_{\text{up}} = \frac{10 \text{ m s}^{-1}}{9.81 \text{ m s}^{-2}} = 1.02 \text{ s}$$

- total time in the air $= 2 \times 1.02$ s $= \mathbf{2.0}$ s

(c) $u_H = 20$ m s$^{-1} \times \cos 30° = 17.3$ m s^{-1}

- range $= u_H \times t = (17.3 \text{ m s}^{-1})(2.04 \text{ s}) = \mathbf{35}$ m

3.5

(a) same start time and positive direction

- car (constant v): $s = (18 \text{ m s}^{-1})t$
- motorcyclist (from rest): $s = \frac{1}{2}(3.0 \text{ m s}^{-2})t^2 = (1.5 \text{ m s}^{-2})t^2$
- they meet when the displacements are equal

$$(18 \text{ m s}^{-1})t = (1.5 \text{ m s}^{-2})t^2 \Rightarrow t = \frac{18 \text{ m s}^{-1}}{1.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

(b) $s = (18 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{216}$ m

(c) $v = at = (3.0 \text{ m s}^{-2})(12 \text{ s}) = \mathbf{36}$ m s^{-1}

3.6

(a) acceleration is the **rate of change of velocity**

(b) a = gradient of the v–t graph

$$a = \frac{24 \text{ m s}^{-1} - 0}{8.0 \text{ s}} = \mathbf{3.0} \text{ m s}^{-2}$$

(c) height = area under the line (a triangle)

$$s = \tfrac{1}{2} \times 8.0 \text{ s} \times 24 \text{ m s}^{-1} = \mathbf{96} \text{ m}$$

(d)

$$\Delta E_{\text{P}} = mgh = (1.8 \text{ kg})(9.81 \text{ m s}^{-2})(96 \text{ m}) = \mathbf{1.7 \times 10^3} \text{ J}$$

(e)

$$\Delta E_{\text{K}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.8 \text{ kg})(24 \text{ m s}^{-1})^2 = \mathbf{5.2 \times 10^2} \text{ J}$$

(f) useful energy gained $= \Delta E_{\text{P}} + \Delta E_{\text{K}}$

$$P = \frac{1.7 \times 10^3 \text{ J} + 5.2 \times 10^2 \text{ J}}{8.0 \text{ s}} = \mathbf{2.8 \times 10^2} \text{ W}$$

(g)

- axes drawn and labelled displacement / m (vertical) and time / s (horizontal)
- a curve starting at the origin
- getting **steeper** with time ($s \propto t^2$), reaching 96 m at 8.0 s

(h) **greater**

- the v–t graph is the same, so Δh and Δv are the same, but $\Delta E_{\text{P}} = mgh$ and $\Delta E_{\text{K}} = \frac{1}{2}mv^2$ both scale with **mass**, so more energy is gained in the same 8.0 s