A-LEVEL PHYSICS • EXERCISE SHEET

2.1.5 Projectile motion

Name: __________ Class: ________ Date: _________ **Total: 37 marks**

KEY WORDS projectile 抛体 • component 分量 • range 射程 • horizontal 水平 • velocity 速度

Objective

Build the skills to answer exam questions on **projectile motion 抛体运动**: an object with a constant velocity in one direction and a constant acceleration at right angles to it. This sheet covers syllabus 2.1, learning outcome 9.

You must be able to:

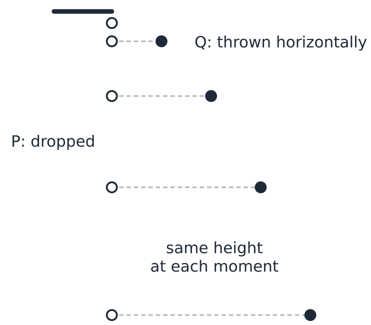
- explain why the horizontal and vertical motions of a **projectile 抛体** are independent, and linked only by the time
- solve a horizontal launch: the time of flight from the height, then the distance from the horizontal velocity
- **resolve 分解** an angled launch velocity into horizontal and vertical **components 分量**
- find the time of flight, the greatest height and the **range 射程**
- give the velocity at any moment as a size and a direction
- sketch how the horizontal and vertical components of the velocity change with time

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Two independent motions

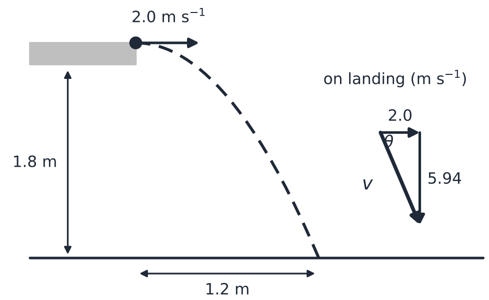
Ball P is dropped from rest. At the same moment, ball Q is thrown **horizontally 水平** from the same height. Air resistance is negligible. The figure shows both balls at equal time intervals.



- vertically, both balls start with zero vertical velocity and accelerate at g, so they are at the same height at every moment and land together
- horizontally, nothing changes Q's horizontal velocity, so it moves the same distance in each interval
- the two motions share only the time t: find t from the vertical motion, then use it in the horizontal motion

■ A horizontal launch

A ball rolls off a horizontal shelf 1.8 m above the floor at 2.0 m s^{-1}.



- vertical motion: $u_{\mathrm{V}} = 0$, $s = 1.8$ m and $a = 9.81$ m s^{-2}, taking down as positive

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1.8\ \mathrm{m}}{9.81\ \mathrm{m\ s^{-2}}}} = 0.606\ \mathrm{s}$$

- horizontal motion: constant velocity

$$x = (2.0\ \mathrm{m\ s^{-1}})(0.606\ \mathrm{s}) = 1.2\ \mathrm{m}$$

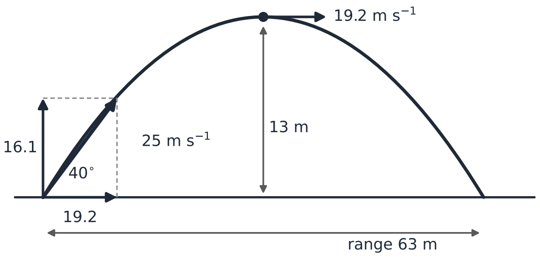
- on landing, $v_H = 2.0$ m s^{-1} and $v_V = at = (9.81 \text{ m s}^{-2})(0.606 \text{ s}) = 5.94$ m s^{-1}
- add the components like any two vectors at right angles

$$v = \sqrt{(2.0 \text{ m s}^{-1})^2 + (5.94 \text{ m s}^{-1})^2} = 6.3 \text{ m s}^{-1}, \qquad \tan\theta = \frac{5.94}{2.0} \Rightarrow \theta = 71^\circ$$

- the velocity is 6.3 m s^{-1} at 71° below the horizontal

■ An angled launch

A ball is kicked from level ground at 25 m s^{-1} at 40° above the horizontal.



- resolve the launch velocity

$$u_H = (25 \text{ m s}^{-1})\cos 40^\circ = 19.2 \text{ m s}^{-1}, \qquad u_V = (25 \text{ m s}^{-1})\sin 40^\circ = 16.1 \text{ m s}^{-1}$$

- take up as positive; at the top $v_V = 0$

$$t_{top} = \frac{0 - 16.1 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}} = 1.64 \text{ s}, \qquad H = \frac{0 - (16.1 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = 13 \text{ m}$$

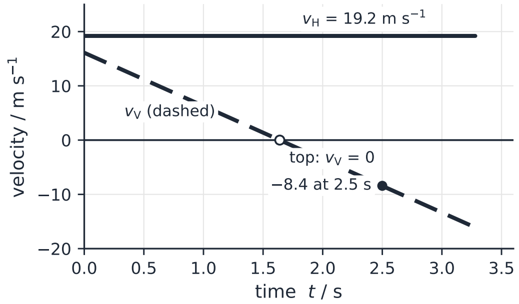
- it lands at the same level, so the whole flight takes twice as long, 3.28 s

$$R = (19.2 \text{ m s}^{-1})(3.28 \text{ s}) = 63 \text{ m}$$

- at the top the velocity is **not** zero: it is 19.2 m s^{-1} horizontally

■ Velocity at any moment

For the same ball at $t = 2.5$ s, up positive:



- the horizontal component is unchanged: $v_H = 19.2$ m s^{-1}
- the vertical component

$$v_V = u_V + at = 16.1 \text{ m s}^{-1} + (-9.81 \text{ m s}^{-2})(2.5 \text{ s}) = -8.4 \text{ m s}^{-1}$$

- size and direction

$$v = \sqrt{(19.2 \text{ m s}^{-1})^2 + (8.4 \text{ m s}^{-1})^2} = 21 \text{ m s}^{-1}, \qquad \tan\theta = \frac{8.4}{19.2} \Rightarrow \theta = 24^\circ \text{ below the horizontal}$$

- on the graph, v_H is a horizontal line and v_V is a straight line of gradient -9.81 m s^{-2} that crosses zero at the top

2 Practice

Now use the methods above.

2.1 Two identical balls leave the edge of a horizontal table at the same moment. Ball X falls from rest and ball Y is pushed off horizontally. Air resistance is negligible. State which ball reaches the floor first, and explain your answer. [2]

2.2 A delivery drone flies horizontally at 12 m s^{-1}, 45 m above flat ground, and releases a parcel. Air resistance is negligible.

(a) State the horizontal and vertical components of the parcel's velocity just after it is released. [1]

(b) Calculate the time taken for the parcel to reach the ground. [2]

(c) Calculate the horizontal distance the parcel travels. [1]

(d) Calculate the speed of the parcel as it reaches the ground. [2]

2.3 A football is kicked from level ground at 18 m s^{-1} at 35° above the horizontal. Air resistance is negligible.

(a) Calculate the horizontal and vertical components of the initial velocity. [2]

(b) Calculate the time taken for the ball to reach its greatest height. [1]

(c) Calculate the greatest height. [2]

(d) Calculate the range. [2]

2.4 For the football in **2.3**, sketch on one set of axes how the horizontal and vertical components of its velocity change with time, from the kick until it lands. Take up as positive, label each line, and mark the values at the start and at the end. [3]

3 Exam-style questions

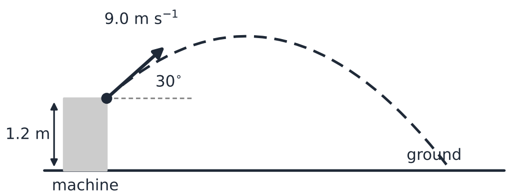
3.1 A ball is thrown at an angle above horizontal ground. Air resistance is negligible. Which row describes the ball at the highest point of its path? [1]

	velocity	acceleration
A	zero	zero
B	zero	$9.81\ \mathrm{m\ s^{-2}}$ downwards
C	horizontal	zero
D	horizontal	$9.81\ \mathrm{m\ s^{-2}}$ downwards

3.2 A ball is thrown horizontally from the top of a building 20 m high. It lands 30 m from the base of the building. Air resistance is negligible. What was its initial speed? [1]

- **A** $7.4\ \mathrm{m\ s^{-1}}$
- **B** $15\ \mathrm{m\ s^{-1}}$
- **C** $20\ \mathrm{m\ s^{-1}}$
- **D** $21\ \mathrm{m\ s^{-1}}$

3.3 A ball-launching machine is 1.2 m above level ground. It fires a ball at $9.0\ \mathrm{m\ s^{-1}}$ at 30° above the horizontal. Air resistance is negligible.



(a) Calculate the horizontal and vertical components of the ball's initial velocity. [2]

(b) Calculate the greatest height of the ball above the ground. [2]

(c) **Show that** the ball takes about 1.1 s to reach the ground. [3]

(d) Calculate the horizontal distance from the machine to where the ball lands. [1]

(e) Calculate the speed of the ball as it lands, and the angle its velocity makes with the ground. [3]

3.4 A marble rolls off the edge of a horizontal table 0.80 m high. It lands on the floor 0.52 m from the foot of the table. Air resistance is negligible.

(a) **Show that** the marble takes 0.40 s to fall to the floor. [2]

(b) Calculate the speed of the marble as it leaves the table. [1]

(c) A second marble leaves the same table at twice the speed. State the time it takes to reach the floor and how far from the table it lands. Explain your answers. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- they reach the floor **at the same time**
- both start with zero vertical velocity, fall the same height and have the same acceleration g; the horizontal velocity of Y does not change its vertical motion

2.2

(a) horizontal 12 m s^{-1} (the drone's velocity) and vertical 0

(b) vertical motion from rest, down positive

$$t = \sqrt{\frac{2 \times 45 \text{ m}}{9.81 \text{ m s}^{-2}}} = \mathbf{3.0} \text{ s}$$

(c) $(12 \text{ m s}^{-1})(3.03 \text{ s}) = \mathbf{36} \text{ m}$

(d) $v_V = (9.81 \text{ m s}^{-2})(3.03 \text{ s}) = 29.7 \text{ m s}^{-1}$

$$v = \sqrt{(12 \text{ m s}^{-1})^2 + (29.7 \text{ m s}^{-1})^2} = \mathbf{32} \text{ m s}^{-1}$$

2.3

(a) resolve

$$u_H = (18 \text{ m s}^{-1}) \cos 35° = \mathbf{14.7} \text{ m s}^{-1}, \qquad u_V = (18 \text{ m s}^{-1}) \sin 35° = \mathbf{10.3} \text{ m s}^{-1}$$

(b) at the top $v_V = 0$

$$t = \frac{0 - 10.3 \text{ m s}^{-1}}{-9.81 \text{ m s}^{-2}} = \mathbf{1.05} \text{ s}$$

(c) use $v^2 = u^2 + 2as$ vertically

$$H = \frac{0 - (10.3 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = \mathbf{5.4} \text{ m}$$

(d) time of flight $= 2 \times 1.05 \text{ s} = 2.10 \text{ s}$

$$R = (14.7 \text{ m s}^{-1})(2.10 \text{ s}) = \mathbf{31} \text{ m}$$

2.4

- horizontal component: a horizontal line at 14.7 m s^{-1} from 0 to 2.1 s
- vertical component: a straight line from +10.3 m s^{-1} at $t = 0$
- through zero at 1.05 s to −10.3 m s^{-1} at 2.1 s, with both lines labelled

3.1 D

- at the top the vertical component is zero, but the horizontal component is not, so the velocity is horizontal
- the acceleration is g downwards during the whole flight, including at the top
- **A** and **B** forget the horizontal velocity; **C** thinks the acceleration stops at the top

3.2 B

- vertical motion from rest gives the time

$$t = \sqrt{\frac{2 \times 20 \text{ m}}{9.81 \text{ m s}^{-2}}} = 2.02 \text{ s}$$

- horizontal motion gives the speed

$$u = \frac{30 \text{ m}}{2.02 \text{ s}} = \mathbf{15} \text{ m s}^{-1}$$

- **A** doubles the time; **C** is the vertical speed on landing; **D** leaves out the 2 in $s = \frac{1}{2}at^2$

3.3

(a) resolve

$$u_{\text{H}} = (9.0 \text{ m s}^{-1}) \cos 30° = \mathbf{7.8} \text{ m s}^{-1}, \qquad u_{\text{V}} = (9.0 \text{ m s}^{-1}) \sin 30° = \mathbf{4.5} \text{ m s}^{-1}$$

(b) rise above the machine, with $v_{\text{V}} = 0$ at the top

$$\frac{0 - (4.5 \text{ m s}^{-1})^2}{2(-9.81 \text{ m s}^{-2})} = 1.03 \text{ m}$$

- height above the ground $= 1.03 \text{ m} + 1.2 \text{ m} = \mathbf{2.2}$ m

(c) up positive: $s = -1.2$ m, $u = +4.5$ m s^{-1}, $a = -9.81$ m s^{-2}

$$-1.2 = 4.5t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.5t - 1.2 = 0$$

- quadratic formula, values in m and s

$$t = \frac{4.5 + \sqrt{20.25 + 23.5}}{9.81} = 1.13 \text{ s} \approx 1.1 \text{ s}$$

(d) $(7.8 \text{ m s}^{-1})(1.13 \text{ s}) = \mathbf{8.8}$ m

(e) $v_{\text{V}} = 4.5 \text{ m s}^{-1} + (-9.81 \text{ m s}^{-2})(1.13 \text{ s}) = -6.6 \text{ m s}^{-1}$

$$v = \sqrt{(7.8 \text{ m s}^{-1})^2 + (6.6 \text{ m s}^{-1})^2} = \mathbf{10} \text{ m s}^{-1}$$

- $\tan\theta = 6.6/7.8$, so the velocity is at **40**° below the horizontal

3.4

(a) vertical motion from rest

$$t = \sqrt{\frac{2 \times 0.80 \text{ m}}{9.81 \text{ m s}^{-2}}} = 0.404 \text{ s} \approx 0.40 \text{ s}$$

(b)

$$u = \frac{0.52 \text{ m}}{0.404 \text{ s}} = \mathbf{1.3} \text{ m s}^{-1}$$

(c)

- it still takes **0.40** s
- the vertical motion is unchanged: zero vertical velocity at the start, the same height and the same g
- it lands **1.0** m from the table, because the horizontal distance is the horizontal speed multiplied by the same time