A-LEVEL PHYSICS • EXERCISE SHEET

2.1.4 Free fall and measuring g

Name: ______________ Class: __________ Date: ____________ **Total: 38 marks**

KEY WORDS air resistance 空气阻力 • acceleration 加速度 • free fall 自由落体 • acceleration of free fall 自由落体加速度 • velocity 速度

Objective

Build the skills to answer exam questions on **free fall 自由落体**: vertical motion under gravity with one sign convention, the velocity–time graph of a thrown or bouncing ball, and the experiment that measures the **acceleration of free fall 自由落体加速度** g. This sheet covers syllabus 2.1, learning outcomes 7 (vertical motion) and 8.

You must be able to:

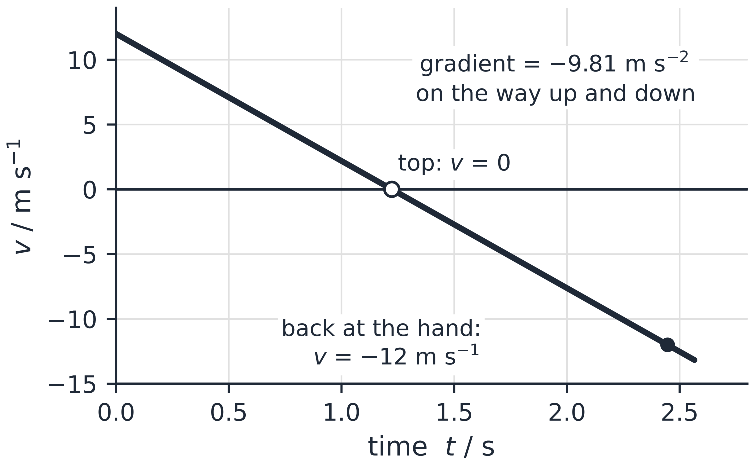
- use $g = 9.81$ m s^{-2} with a sign convention for an object that goes up and comes down
- find the greatest height, the time to reach it, and the time to land
- give an object released from a moving balloon its correct initial velocity
- draw and read the velocity–time graph of a thrown or bouncing ball
- **describe** an experiment to measure g, and find g from the gradient of a graph
- estimate the uncertainty in g, and explain the effect of a systematic error

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Up and down with one sign convention

A ball is thrown vertically upwards at 12 m s^{-1} from a hand 1.5 m above the ground. **Air resistance 空气阻力** is negligible. Take **up as positive**, so $u = +12$ m s^{-1} and $a = -9.81$ m s^{-2} for the whole flight, on the way up and on the way down.



- greatest height above the hand: at the top $v = 0$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (12\ \text{m s}^{-1})^2}{2(-9.81\ \text{m s}^{-2})} = 7.3\ \text{m}$$

- time to reach the top

$$t = \frac{v - u}{a} = \frac{0 - 12\ \text{m s}^{-1}}{-9.81\ \text{m s}^{-2}} = 1.2\ \text{s}$$

- the ground is 1.5 m **below** the hand, so $s = -1.5$ m (values in m and s)

$$-1.5 = 12t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 12t - 1.5 = 0 \quad \Rightarrow \quad t = 2.6\ \text{s}$$

- the other root of the quadratic is negative, a time before the throw, so it is rejected
- velocity on landing

$$v^2 = (12\ \text{m s}^{-1})^2 + 2(-9.81\ \text{m s}^{-2})(-1.5\ \text{m}) \quad \Rightarrow \quad v = -13\ \text{m s}^{-1}$$

- the minus sign means the ball is moving downwards

■ Released from something moving

A sandbag is released from a balloon that is rising at 4.0 m s^{-1}, 30 m above the ground. At the moment of release the sandbag moves with the balloon, so $u = +4.0$ m s^{-1}, **not zero**. Take up as positive: $s = -30$ m and $a = -9.81$ m s^{-2}.

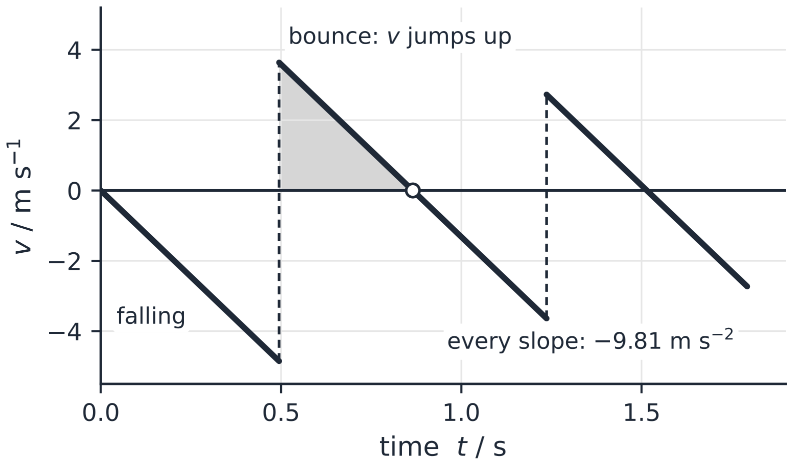
- time to reach the ground (values in m and s)

$$-30 = 4.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 4.0t - 30 = 0 \quad \Rightarrow \quad t = 2.9\ \text{s}$$

- taking $u = 0$ gives 2.5 s, which is wrong: the sandbag rises a little before it falls

■ The velocity–time graph of a bouncing ball

A ball is dropped from rest from a height of 1.2 m and bounces. Up is positive.

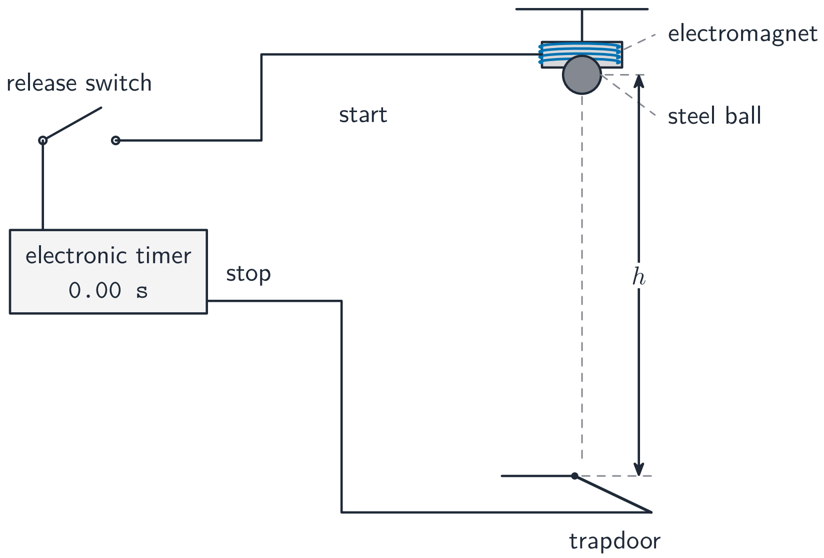


- every sloping line has the same gradient, -9.81 m s^{-2}: while the ball is in the air its acceleration is g downwards
- at a bounce the velocity jumps from negative (moving down) to positive (moving up), and it is smaller after the bounce
- the ball is at the top of a bounce where the line crosses the time axis, $v = 0$
- the height of the bounce is the area of the triangle above the axis

$$\tfrac{1}{2} \times 0.37\ \text{s} \times 3.6\ \text{m s}^{-1} = 0.67\ \text{m}$$

■ Measuring g

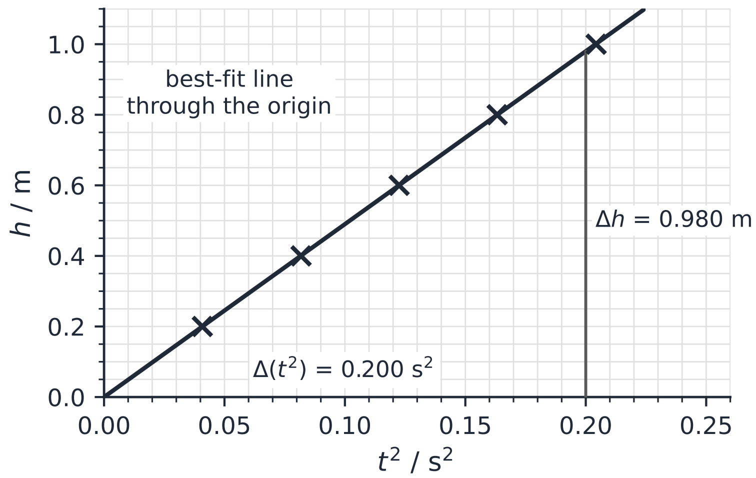
The syllabus asks you to **describe** this experiment.



1. Measure the height h from the bottom of the steel ball to the trapdoor with a metre rule.
2. Switching off the electromagnet releases the ball and starts the electronic timer. The ball opens the trapdoor, which stops the timer, so the timer shows the fall time t.

3. Repeat each timing and find the mean. Do this for at least five different heights.
4. The ball starts from rest, so $h = \frac{1}{2}gt^2$. Plot h against t^2: the gradient is $\frac{1}{2}g$.

h / m	0.200	0.400	0.600	0.800	1.000
t / s	0.202	0.286	0.350	0.404	0.452
t^2 / s^2	0.0408	0.0818	0.1225	0.1632	0.2043



- gradient of the best-fit line, from the large triangle

$$\text{gradient} = \frac{0.980\ \text{m}}{0.200\ \text{s}^2} = 4.90\ \text{m s}^{-2}$$

- $g = 2 \times \text{gradient} = 2 \times 4.90\ \text{m s}^{-2} = 9.80\ \text{m s}^{-2}$

Uncertainty. $g = 2h/t^2$, so the percentage uncertainties add, and the time counts twice because it is squared (Topic 1). For $h = 0.800 \pm 0.002$ m and $t = 0.404 \pm 0.004$ s:

$$\frac{\Delta g}{g} = \frac{0.002\ \text{m}}{0.800\ \text{m}} + 2 \times \frac{0.004\ \text{s}}{0.404\ \text{s}} = 0.25\,\% + 2.0\,\% = 2.2\,\%$$

so $g = 9.8 \pm 0.2\ \text{m s}^{-2}$.

Systematic error. If the ball is released a short time **after** the timer starts, every measured time is too long by the same amount. Each calculated value of g is then too small, and the graph of h against t^2 no longer passes through the origin.

2 Practice

Now use the methods above.

2.1 A stone is dropped from rest from a bridge 20 m above a river. Air resistance is negligible.

(a) Calculate the time taken for the stone to reach the water. [2]

(b) Calculate the speed of the stone as it reaches the water. [2]

2.2 A ball is thrown vertically upwards at 18 m s^{-1} and is caught at the same height. Take up as positive.

(a) Calculate the greatest height the ball reaches. [2]

(b) Calculate the total time the ball is in the air. [2]

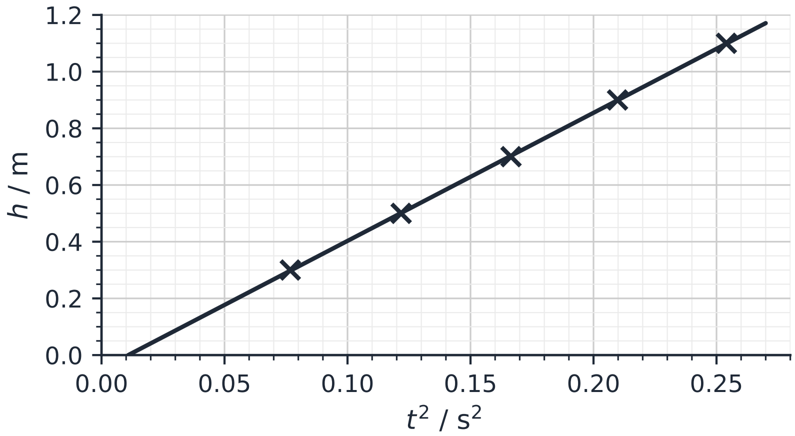
(c) State the velocity of the ball just before it is caught. [1]

2.3 A camera falls out of a hot-air balloon that is rising at 3.0 m s^{-1}. At that moment the camera is 45 m above the ground.

(a) State the velocity of the camera at the moment it leaves the balloon. [1]

(b) Calculate the time taken for the camera to reach the ground. *(Hint: with up as positive, $s = -45$ m.)* [3]

2.4 A student measures the time t for a ball to fall from rest through different heights h. The graph shows her results and her best-fit line.



(a) Determine the gradient of the line. [2]

(b) Use your answer to (a) to find a value for g. [1]

(c) The line does not pass through the origin. Suggest one reason. [1]

3 Exam-style questions

3.1 An object falls from rest. Air resistance is negligible. How far does it fall during the third second of its fall, from $t = 2.0$ s to $t = 3.0$ s? [1]

- **A** 9.8 m
- **B** 25 m
- **C** 29 m
- **D** 44 m

3.2 A ball is dropped from rest through different heights h, and its speed v at a light gate is measured each time. A graph of v^2 against h is a straight line through the origin with gradient 19.4 m s^{-2}. What is g? [1]

- **A** 4.9 m s^{-2}
- **B** 9.7 m s^{-2}
- **C** 19 m s^{-2}
- **D** 39 m s^{-2}

3.3 A student has a steel ball, an electromagnet, a trapdoor switch, an electronic timer and a metre rule. She uses them to determine the acceleration of free fall g.

(a) **Describe** how she carries out the experiment. Include the measurements she takes and how she finds g from a graph. [4]

(b) For one height, $h = 0.750 \pm 0.002$ m and $t = 0.393 \pm 0.005$ s. Calculate g. [2]

(c) Calculate the percentage uncertainty in her value of g, and hence give g with its absolute uncertainty. [3]

(d) The electromagnet takes a short time to release the ball after the timer starts. State and explain the effect of this on the value of g calculated from one measurement. [2]

3.4 A ball is thrown vertically upwards at 8.0 m s^{-1} from the top of a cliff. It then falls past the cliff top into the sea 25 m below. Air resistance is negligible. Take up as positive.

(a) Calculate the greatest height of the ball above the cliff top. [2]

(b) Calculate the time from the throw until the ball reaches the sea. [3]

(c) **Sketch** the velocity–time graph of the ball from the throw until it reaches the sea. Mark the values of the velocity at the start and at the end. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) $u = 0$, $s = 20$ m, $a = 9.81$ m s^{-2} (down positive); use $s = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 20\ \text{m}}{9.81\ \text{m s}^{-2}}} = \mathbf{2.0}\ \text{s}$$

(b) use $v^2 = u^2 + 2as$

$$v = \sqrt{2 \times 9.81\ \text{m s}^{-2} \times 20\ \text{m}} = \mathbf{20}\ \text{m s}^{-1}$$

2.2

(a) at the top $v = 0$

$$s = \frac{0 - (18\ \text{m s}^{-1})^2}{2(-9.81\ \text{m s}^{-2})} = \mathbf{17}\ \text{m}$$

(b) time to the top, then double it: the flight is symmetrical

$$t_{\text{up}} = \frac{0 - 18\ \text{m s}^{-1}}{-9.81\ \text{m s}^{-2}} = 1.83\ \text{s}, \qquad t = 2 \times 1.83\ \text{s} = \mathbf{3.7}\ \text{s}$$

(c) $-\mathbf{18}$ m s^{-1}: the same speed as the throw, now downwards

2.3

(a) **3.0** m s^{-1} **upwards**, the velocity of the balloon

(b) $u = +3.0$ m s^{-1}, $a = -9.81$ m s^{-2}, $s = -45$ m in $s = ut + \frac{1}{2}at^2$

$$-45 = 3.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 3.0t - 45 = 0$$

- quadratic formula (values in m and s)

$$t = \frac{3.0 + \sqrt{9.0 + 883}}{9.81} = \mathbf{3.4}\ \text{s}$$

2.4

(a) large triangle on the line, for example from (0.050 s^2, 0.18 m) to (0.250 s^2, 1.08 m)

$$\text{gradient} = \frac{1.08\ \text{m} - 0.18\ \text{m}}{0.250\ \text{s}^2 - 0.050\ \text{s}^2} = \mathbf{4.5}\ \text{m s}^{-2}$$

(accept 4.4 to 4.6)

(b) $g = 2 \times \text{gradient} = \mathbf{9.0}$ m s^{-2} (accept 8.8 to 9.2)

(c) any one: every time reading is too long by the same amount, for example the ball is released a short time after the timer starts; or h is measured from the wrong point on the ball

3.1 B

- displacement after 3.0 s minus displacement after 2.0 s

$$\tfrac{1}{2}(9.81\ \text{m s}^{-2})(3.0\ \text{s})^2 - \tfrac{1}{2}(9.81\ \text{m s}^{-2})(2.0\ \text{s})^2 = 44.1\ \text{m} - 19.6\ \text{m} = \mathbf{25}\ \text{m}$$

- **A** is $g \times 1$ s; **C** is the speed at 3.0 s; **D** is the whole distance in 3.0 s

3.2 B

- from $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2gh$, so the gradient is $2g$

$$g = \frac{19.4\ \text{m s}^{-2}}{2} = \mathbf{9.7}\ \text{m s}^{-2}$$

- **C** takes the gradient as g; **A** and **D** use the wrong factor of 2

3.3

(a)

- measure h from the bottom of the ball to the trapdoor with the metre rule
- switching off the electromagnet releases the ball and starts the timer; the ball hitting the trapdoor stops it, giving t
- repeat for several different heights, repeating each timing and taking the mean
- plot h against t^2 and find the gradient; $g = 2\times$ gradient

(b) the ball starts from rest, so $h = \frac{1}{2}gt^2$

$$g = \frac{2h}{t^2} = \frac{2 \times 0.750\ \text{m}}{(0.393\ \text{s})^2} = \mathbf{9.71}\ \text{m s}^{-2}$$

(c) percentage uncertainties

$$\frac{0.002\ \text{m}}{0.750\ \text{m}} = 0.27\,\%, \qquad \frac{0.005\ \text{s}}{0.393\ \text{s}} = 1.27\,\%$$

- the time is squared, so it counts twice: $0.27\,\% + 2 \times 1.27\,\% = \mathbf{2.8}\,\%$
- $2.8\,\%$ of 9.71 m s^{-2} is 0.27 m s^{-2}, so $g = \mathbf{9.7} \pm \mathbf{0.3}$ m s^{-2}

(d)

- the measured time is longer than the real time of fall
- $g = 2h/t^2$, so the calculated value of g is too small

3.4

(a) at the top $v = 0$

$$s = \frac{0 - (8.0\ \text{m s}^{-1})^2}{2(-9.81\ \text{m s}^{-2})} = \mathbf{3.3}\ \text{m}$$

(b) $s = -25$ m, $u = +8.0$ m s^{-1}, $a = -9.81$ m s^{-2}

$$-25 = 8.0t - 4.905t^2 \quad \Rightarrow \quad 4.905t^2 - 8.0t - 25 = 0$$

- quadratic formula, keeping the positive root

$$t = \frac{8.0 + \sqrt{64 + 490.5}}{9.81} = \mathbf{3.2} \text{ s}$$

(c)

- a single straight line with a negative gradient
- starting at $+8.0$ m s^{-1} and crossing $v = 0$ at about 0.8 s
- ending at about -24 m s^{-1} at 3.2 s