A-LEVEL PHYSICS • EXERCISE SHEET

2.1.3 The equations of uniformly accelerated motion

Name: ____________ Class: ________ Date: __________ **Total: 37 marks**

KEY WORDS uniform acceleration 匀加速 • velocity 速度 • deceleration 减速度 • acceleration 加速度 • speed 速率

Objective

Build the skills to answer exam questions with the **equations of uniformly accelerated motion** 匀加速运动方程: deriving them, choosing the right one, using signs, and solving problems with two stages or two objects. This sheet covers syllabus 2.1, learning outcomes 6 and 7 (motion in a straight line).

You must be able to:

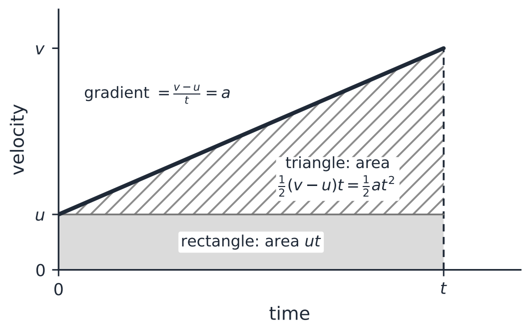
- **derive** 推导 the equations from the definition of acceleration and the area under a velocity–time graph
- state when the equations can be used
- choose the equation that links what you know to what you want
- choose a positive direction and give every vector a sign
- solve problems with two stages, or with two objects
- solve a **quadratic** 二次方程 with two answers, and decide which answers make sense

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

Deriving the equations

An object has **uniform acceleration** 匀加速 a in a straight line. Its velocity changes from u to v in a time t, and its displacement is s.



- from the definition of acceleration (the gradient)

$$a = \frac{v-u}{t} \quad \Rightarrow \quad v = u + at$$

- displacement = area under the line, a **trapezium** 梯形

$$s = \tfrac{1}{2}(u+v)t$$

- put $v = u + at$ into the trapezium; this is the rectangle plus the triangle in the graph

$$s = \tfrac{1}{2}(u + u + at)t = ut + \tfrac{1}{2}at^2$$

- from the first equation $t = (v-u)/a$; put this into $s = \frac{1}{2}(u+v)t$

$$s = \frac{(v+u)(v-u)}{2a} \quad \Rightarrow \quad v^2 = u^2 + 2as$$

The equations work only when the acceleration is **constant** and the motion is in a **straight line**. $s = \frac{1}{2}at^2$ also needs the object to start from rest ($u = 0$).

Choosing an equation

Write down the five symbols s, u, v, a, t. Find the one you are not given and do not want: use the equation without it.

not given and not wanted	use
s	$v = u + at$
a	$s = \frac{1}{2}(u+v)t$
v	$s = ut + \frac{1}{2}at^2$
t	$v^2 = u^2 + 2as$

Example. A cyclist brakes uniformly from 9.0 m s^{-1} to rest in a distance of 12 m.

- $u = 9.0$ m s^{-1}, $v = 0$, $s = 12$ m; to find a, there is no t

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - (9.0\ \text{m s}^{-1})^2}{2 \times 12\ \text{m}} = -3.4\ \text{m s}^{-2}$$

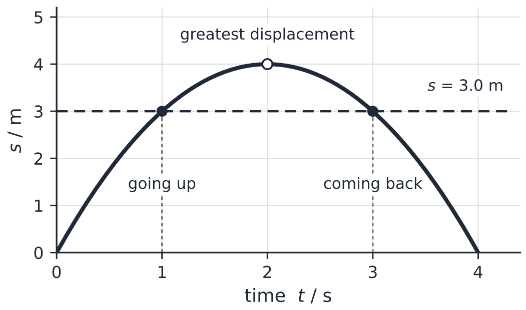
- to find t, there is no a

$$t = \frac{2s}{u+v} = \frac{2 \times 12 \text{ m}}{9.0 \text{ m s}^{-1}} = 2.7 \text{ s}$$

- the minus sign shows the acceleration is opposite to the velocity: a **deceleration** 减速度 of 3.4 m s^{-2}

■ Signs, and two answers

Choose one direction as positive and keep it for the whole problem. A ball rolls up a slope at 4.0 m s^{-1}. Its acceleration is 2.0 m s^{-2} down the slope, on the way up and on the way back down. Take **up the slope as positive**: $u = +4.0$ m s^{-1} and $a = -2.0$ m s^{-2}.



- when does it stop? Use $v = u + at$

$$0 = 4.0 \text{ m s}^{-1} + (-2.0 \text{ m s}^{-2})t \quad \Rightarrow \quad t = 2.0 \text{ s}$$

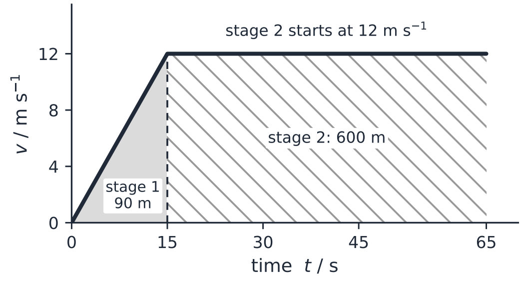
- when is it 3.0 m up the slope? Use $s = ut + \frac{1}{2}at^2$ (values in m and s)

$$3.0 = 4.0t - 1.0t^2 \quad \Rightarrow \quad t^2 - 4.0t + 3.0 = 0 \quad \Rightarrow \quad t = 1.0 \text{ s or } 3.0 \text{ s}$$

- both answers are real: at 1.0 s the ball is going up, and at 3.0 s it is coming back down

■ Two stages

A tram starts from rest with an acceleration of 0.80 m s^{-2} for 15 s. It then keeps a steady speed for 50 s.



- stage 1: the final velocity

$$v = u + at = 0 + (0.80 \text{ m s}^{-2})(15 \text{ s}) = 12 \text{ m s}^{-1}$$

- stage 1: the displacement

$$s_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.80 \text{ m s}^{-2})(15 \text{ s})^2 = 90 \text{ m}$$

- the final velocity of stage 1 is the initial velocity of stage 2

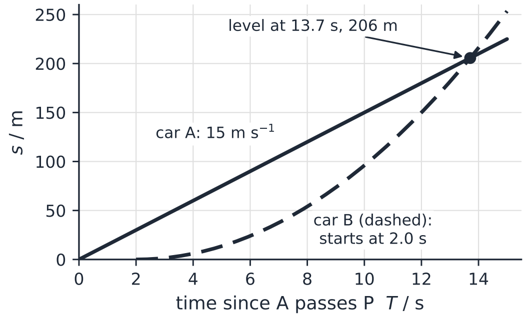
$$s_2 = (12 \text{ m s}^{-1})(50 \text{ s}) = 600 \text{ m}$$

- total displacement = 90 m + 600 m = 690 m

A driver's **reaction time** 反应时间 is a stage too: the car keeps its speed, so the **thinking distance** 反应距离 is speed × reaction time. The **braking distance** 制动距离 comes from $v^2 = u^2 + 2as$, and the **stopping distance** 停车距离 is the sum of the two.

■ Two objects

Car A passes a point P at a constant 15 m s^{-1}. 2.0 s later, car B starts from rest at P with an acceleration of 3.0 m s^{-2}. Measure the time T from when A passes P.



- car A has moved for T and car B for $(T - 2.0)$, so (values in m and s)

$$s_{\mathrm{A}} = 15T, \qquad s_{\mathrm{B}} = \tfrac{1}{2}(3.0)(T - 2.0)^2 = 1.5(T - 2.0)^2$$

- they are level when the displacements are equal

$$15T = 1.5(T - 2.0)^2 \quad \Rightarrow \quad T^2 - 14T + 4.0 = 0 \quad \Rightarrow \quad T = 13.7 \text{ s or } 0.29 \text{ s}$$

- 0.29 s is before B starts, so it is not a meeting: B catches A at $T = 13.7$ s

$$s = (15 \text{ m s}^{-1})(13.7 \text{ s}) = 206 \text{ m}$$

2 Practice

Now use the methods above.

2.1 Show that $s = ut + \frac{1}{2}at^2$ follows from $v = u + at$ and $s = \frac{1}{2}(u + v)t$. [2]

2.2 The equations of uniformly accelerated motion can only be used under certain conditions.

(a) State two conditions that must both be true. [2]

(b) State the extra condition needed to use $s = \frac{1}{2}at^2$. [1]

2.3 Choose the right equation each time.

(a) $u = 2.0$ m s^{-1}, $a = 1.5$ m s^{-2}, $t = 4.0$ s. Find v. [1]

(b) $u = 0$, $v = 12$ m s^{-1}, $s = 18$ m. Find a. [1]

(c) $u = 5.0$ m s^{-1}, $v = 15$ m s^{-1}, $t = 4.0$ s. Find s. [1]

(d) $u = 3.0$ m s^{-1}, $a = 2.0$ m s^{-2}, $s = 10$ m. Find v. [1]

2.4 A car is travelling at 25 m s^{-1} when the driver sees a hazard. The driver's reaction time is 0.60 s. The car then brakes with a constant deceleration of 6.25 m s^{-2}.

(a) Calculate the thinking distance. [1]

(b) Calculate the braking distance. [2]

(c) State the stopping distance. [1]

2.5 A puck slides up a smooth icy ramp at 6.0 m s^{-1}. Its acceleration is 1.5 m s^{-2} down the ramp, on the way up and on the way back down. Take up the ramp as positive.

(a) Calculate the time taken for the puck to stop. [1]

(b) Calculate the greatest distance the puck travels up the ramp. [2]

(c) Calculate the two times at which the puck is 10 m up the ramp. *(Hint: you will need to solve a quadratic.)* [3]

3 Exam-style questions

3.1 A car moving at 5.0 m s^{-1} accelerates uniformly. In the next 4.0 s it travels 44 m. What is its acceleration? [1]

- **A** 1.5 m s^{-2}
- **B** 3.0 m s^{-2}
- **C** 5.5 m s^{-2}
- **D** 6.0 m s^{-2}

3.2 A lift starts from rest and accelerates uniformly at 1.2 m s^{-2} for 2.5 s. It then moves at a constant velocity for 6.0 s, and finally slows uniformly to rest in 2.0 s. How far does the lift travel? [1]

- **A** 18 m
- **B** 25 m
- **C** 32 m
- **D** 33 m

3.3 A delivery van travels along a straight road at a constant velocity of 16 m s^{-1}. It passes a police car that is at rest. 3.0 s later, the police car sets off after the van. It accelerates uniformly at 2.5 m s^{-2} until it reaches 30 m s^{-1}, and then keeps this velocity.

(a) Calculate the time the police car takes to reach 30 m s^{-1}. [1]

(b) Calculate the distance the police car travels while it is accelerating. [2]

(c) **Show that** the police car has not caught the van when it reaches 30 m s^{-1}. [2]

(d) Determine the time, measured from when the van passes the police car, at which the police car catches the van. [3]

(e) On one set of axes, **sketch** the velocity–time graphs of the van and of the police car up to the moment the police car catches the van. Label each line. [2]

3.4 The equation $v^2 = u^2 + 2as$ links velocity and displacement.

(a) Starting from $v = u + at$ and $s = \frac{1}{2}(u+v)t$, **derive** $v^2 = u^2 + 2as$. [3]

(b) A skateboarder accelerates uniformly from rest to 9.0 m s^{-1} over a distance of 18 m. Calculate her acceleration. [2]

(c) Explain why this equation could not be used if her acceleration decreased as she sped up. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- substitute $v = u + at$ into $s = \frac{1}{2}(u+v)t$

$$s = \tfrac{1}{2}(u + u + at)t = \tfrac{1}{2}(2u + at)t$$

- multiply out the bracket: $s = ut + \frac{1}{2}at^2$

2.2

(a) the acceleration is constant; the motion is in a straight line

(b) the object starts from rest, so $u = 0$

2.3

(a) no s, so $v = u + at = 2.0\text{ m s}^{-1} + (1.5\text{ m s}^{-2})(4.0\text{ s}) = \mathbf{8.0}\text{ m s}^{-1}$

(b) no t, so use $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(12\text{ m s}^{-1})^2}{2 \times 18\text{ m}} = \mathbf{4.0}\text{ m s}^{-2}$$

(c) no a, so use $s = \frac{1}{2}(u+v)t$

$$s = \tfrac{1}{2}(5.0\text{ m s}^{-1} + 15\text{ m s}^{-1})(4.0\text{ s}) = \mathbf{40}\text{ m}$$

(d) no t, so use $v^2 = u^2 + 2as$

$$v^2 = (3.0\text{ m s}^{-1})^2 + 2(2.0\text{ m s}^{-2})(10\text{ m}) = 49\text{ m}^2\text{ s}^{-2} \Rightarrow v = \mathbf{7.0}\text{ m s}^{-1}$$

2.4

(a) the speed is constant during the reaction time: $(25\text{ m s}^{-1})(0.60\text{ s}) = \mathbf{15}\text{ m}$

(b) $u = 25\text{ m s}^{-1}$, $v = 0$, $a = -6.25\text{ m s}^{-2}$; use $v^2 = u^2 + 2as$

$$s = \frac{0 - (25\text{ m s}^{-1})^2}{2(-6.25\text{ m s}^{-2})} = \mathbf{50}\text{ m}$$

(c) $15\text{ m} + 50\text{ m} = \mathbf{65}\text{ m}$

2.5

(a) at the top $v = 0$

$$t = \frac{v - u}{a} = \frac{0 - 6.0\text{ m s}^{-1}}{-1.5\text{ m s}^{-2}} = \mathbf{4.0}\text{ s}$$

(b) at the top $v = 0$; use $v^2 = u^2 + 2as$

$$s = \frac{0 - (6.0\text{ m s}^{-1})^2}{2(-1.5\text{ m s}^{-2})} = \mathbf{12}\text{ m}$$

(c) $s = ut + \frac{1}{2}at^2$ with $s = 10$, $u = 6.0$, $a = -1.5$ (values in m and s)

$$10 = 6.0t - 0.75t^2 \quad \Rightarrow \quad 0.75t^2 - 6.0t + 10 = 0$$

- use the quadratic formula

$$t = \frac{6.0 \pm \sqrt{36 - 30}}{1.5}$$

- $t =$ **2.4** s (going up) and $t =$ **5.6** s (coming back down)

3.1 B

- $u = 5.0$ m s^{-1}, $t = 4.0$ s, $s = 44$ m; there is no v, so use $s = ut + \frac{1}{2}at^2$

$$44 \text{ m} = (5.0 \text{ m s}^{-1})(4.0 \text{ s}) + \tfrac{1}{2}a(4.0 \text{ s})^2 \Rightarrow a = \frac{24 \text{ m}}{8.0 \text{ s}^2} = \mathbf{3.0} \text{ m s}^{\mathbf{-2}}$$

- **A** leaves out the $\frac{1}{2}$; **C** leaves out ut; **D** subtracts u from the average speed but does not divide by the time

3.2 B

- top speed: $(1.2 \text{ m s}^{-2})(2.5 \text{ s}) = 3.0 \text{ m s}^{-1}$
- distance = area under the velocity–time graph

$$\tfrac{1}{2}(2.5 \text{ s})(3.0 \text{ m s}^{-1}) + (6.0 \text{ s})(3.0 \text{ m s}^{-1}) + \tfrac{1}{2}(2.0 \text{ s})(3.0 \text{ m s}^{-1}) = \mathbf{25} \text{ m}$$

- **A** is the steady stage only; **C** uses the top speed while the lift speeds up and slows down; **D** uses the top speed for all 11 s

3.3

(a)

$$t = \frac{v - u}{a} = \frac{30 \text{ m s}^{-1} - 0}{2.5 \text{ m s}^{-2}} = \mathbf{12} \text{ s}$$

(b) no a needed once t is known

$$s = \tfrac{1}{2}(u + v)t = \tfrac{1}{2}(0 + 30 \text{ m s}^{-1})(12 \text{ s}) = \mathbf{180} \text{ m}$$

(c) the van has been moving for 3.0 s + 12 s = 15 s

$$s_{\text{van}} = (16 \text{ m s}^{-1})(15 \text{ s}) = 240 \text{ m}$$

- 240 m is more than 180 m, so the van is still ahead

(d) the gap at 15 s is 240 m − 180 m = 60 m

- the police car closes the gap at 30 m s^{-1} − 16 m s^{-1} = 14 m s^{-1}

$$\frac{60 \text{ m}}{14 \text{ m s}^{-1}} = 4.3 \text{ s} \quad \Rightarrow \quad t = 15 \text{ s} + 4.3 \text{ s} = \mathbf{19} \text{ s}$$

(e)

- van: a horizontal line at 16 m s^{-1} from $t = 0$

- police car: zero until 3.0 s, a straight line up to 30 m s^{-1} at 15 s, then horizontal to about 19 s

3.4

(a)

- from $v = u + at$, $t = (v - u)/a$
- substitute into $s = \frac{1}{2}(u + v)t$

$$s = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$$

- so $2as = v^2 - u^2$, which gives $v^2 = u^2 + 2as$

(b) $u = 0$

$$a = \frac{v^2}{2s} = \frac{(9.0 \text{ m s}^{-1})^2}{2 \times 18 \text{ m}} = \mathbf{2.3} \text{ m s}^{\mathbf{-2}}$$

(c) the equation is true only when the acceleration is constant