A-LEVEL PHYSICS • EXERCISE SHEET

2.1.2 Velocity–time and acceleration–time graphs

Name: ________ Class: ______ Date: ________ **Total: 30 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • displacement 位移 • change in velocity 速度变化 • gradient 斜率

Objective

Build the skills to answer exam questions on **velocity–time graphs 速度 – 时间图像** and **acceleration–time graphs 加速度 – 时间图像**: an acceleration from a gradient, a displacement from an area, and one graph drawn from another. This sheet covers syllabus 2.1, learning outcomes 2, 3 and 5.

You must be able to:

- find an acceleration from the gradient of a velocity–time graph, including at one instant on a curve
- find a displacement from the area under a velocity–time graph, including area below the time axis
- tell distance from displacement on a velocity–time graph
- estimate the area under a curve by counting squares
- find a change in velocity from the area under an acceleration–time graph
- **sketch** one motion graph from another

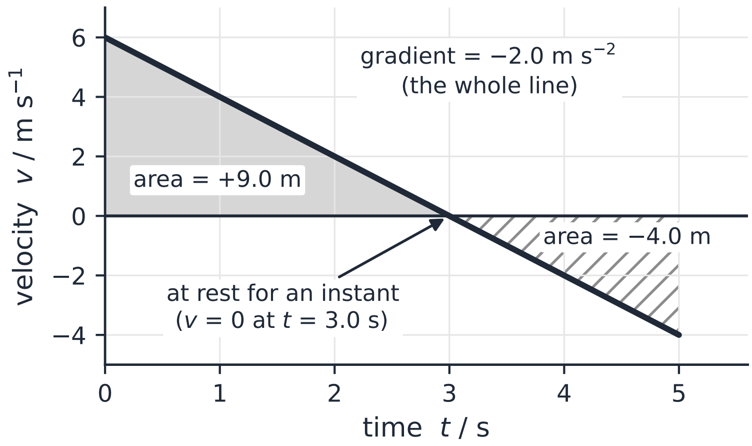
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Gradient and area of a velocity–time graph

On a velocity–time graph the **gradient 斜率** is the **acceleration 加速度**, and the **area 面积** between the line and the time axis is the **displacement 位移**. Area **below** the time axis is negative displacement: the object is moving backwards. The **distance 距离** adds up the sizes of all the areas.

A trolley is pushed up a track at 6.0 m s^{-1}. It slows down, stops for an instant at $t = 3.0$ s, and rolls back.



- acceleration = gradient

$$a = \frac{\Delta v}{\Delta t} = \frac{-4.0\ \text{m s}^{-1} - 6.0\ \text{m s}^{-1}}{5.0\ \text{s}} = -2.0\ \text{m s}^{-2}$$

- the line is straight, so the acceleration is still -2.0 m s^{-2} at $t = 3.0$ s, when the velocity is zero
- area above the axis

$$\tfrac{1}{2} \times 3.0\ \text{s} \times 6.0\ \text{m s}^{-1} = +9.0\ \text{m}$$

- area below the axis

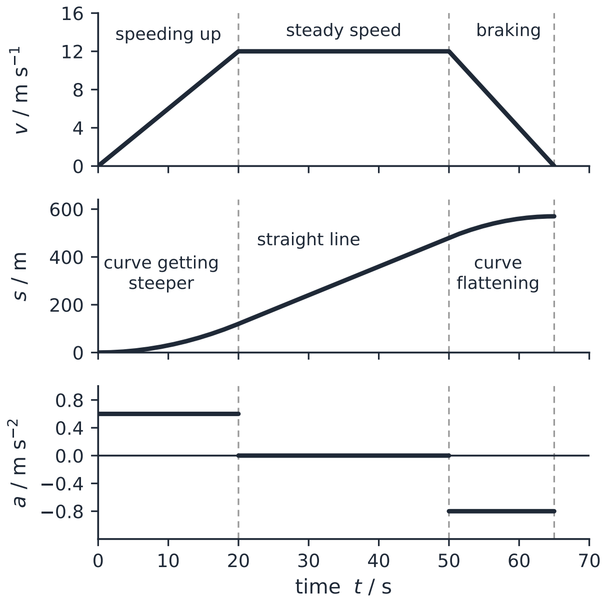
$$\tfrac{1}{2} \times 2.0\ \text{s} \times (-4.0\ \text{m s}^{-1}) = -4.0\ \text{m}$$

- displacement after 5.0 s: 9.0 m − 4.0 m = 5.0 m up the track; distance: 9.0 m + 4.0 m = 13 m

A negative gradient means the acceleration points in the negative direction. On an acceleration–time graph, the area under the line is the **change in velocity** 速度变化.

■ One journey, three graphs

A train starts from rest and speeds up uniformly to 12 m s^{-1} in 20 s. It keeps this speed for 30 s, then brakes uniformly to rest in 15 s.



- **acceleration–time**: each acceleration is a gradient of the velocity–time graph

$$a_1 = \frac{12\ \text{m s}^{-1}}{20\ \text{s}} = 0.60\ \text{m s}^{-2}, \qquad a_3 = \frac{-12\ \text{m s}^{-1}}{15\ \text{s}} = -0.80\ \text{m s}^{-2}$$

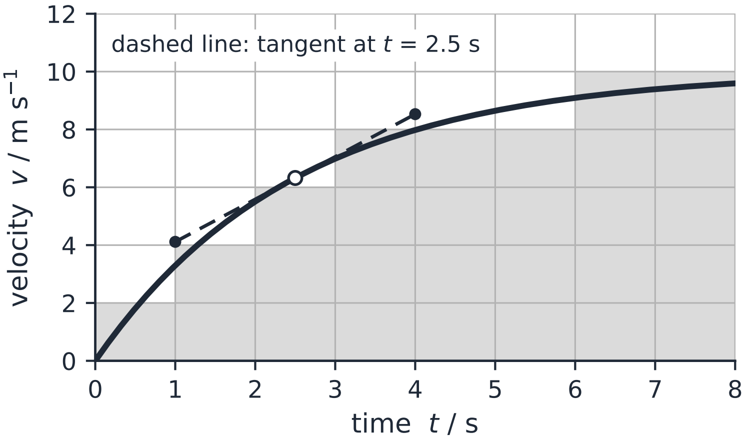
- while the speed is steady, $a = 0$
- **displacement–time**: its gradient is the velocity, so it gets steeper while the train speeds up, is a straight line while the speed is steady, and flattens to horizontal as the train stops
- total displacement = area under the velocity–time graph

$$s = \tfrac{1}{2}(20\ \text{s})(12\ \text{m s}^{-1}) + (30\ \text{s})(12\ \text{m s}^{-1}) + \tfrac{1}{2}(15\ \text{s})(12\ \text{m s}^{-1}) = 570\ \text{m}$$

When you **sketch** 画草图 a graph, draw and label both axes, and give each part the right shape: straight or curved, getting steeper or getting flatter.

■ A curve: a tangent and counting squares

When a velocity–time graph is a curve, the acceleration at one instant is the gradient of the **tangent** 切线 there, and the area is estimated by counting grid squares.



- the tangent at $t = 2.5$ s passes through (1.0 s, 4.1 m s^{-1}) and (4.0 s, 8.5 m s^{-1})

$$a = \frac{8.5\ \text{m s}^{-1} - 4.1\ \text{m s}^{-1}}{4.0\ \text{s} - 1.0\ \text{s}} = 1.5\ \text{m s}^{-2}$$

- one square is 1 s wide and 2 m s^{-1} tall, so it is worth

$$1\ \text{s} \times 2\ \text{m s}^{-1} = 2\ \text{m}$$

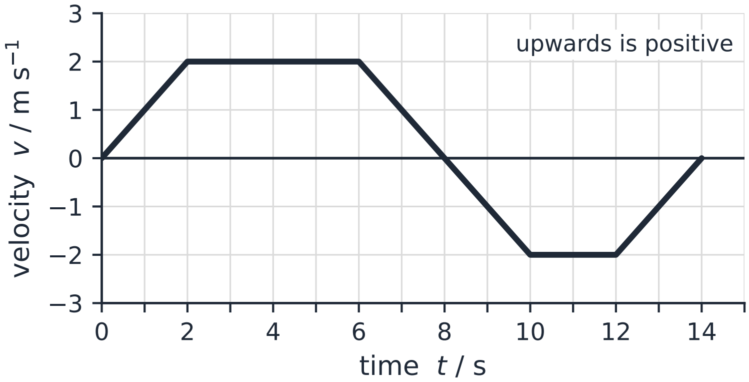
- count every square that is **at least half** under the curve: 28 squares
- displacement $\approx 28 \times 2\ \text{m} = 56\ \text{m}$

2 Practice

Now use the methods above.

2.1 State what is represented by (a) the gradient of a velocity–time graph; (b) the area under a velocity–time graph. [2]

2.2 The velocity–time graph shows a lift that goes up and then comes back down. Upwards is positive.



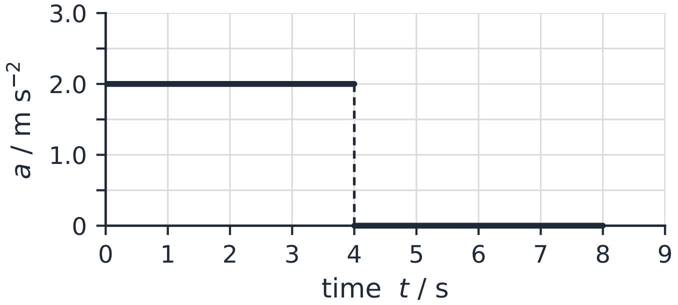
(a) Calculate the acceleration of the lift between $t = 0$ and $t = 2.0$ s. [1]

(b) State the times at which the lift is at rest. [1]

(c) Calculate the total distance travelled by the lift in the 14 s. *(Hint: for distance, every area counts as positive.)* [2]

(d) Determine the displacement of the lift from its starting point at $t = 14$ s. [1]

2.3 A car is moving at 5.0 m s^{-1} at time $t = 0$. The graph shows its acceleration.

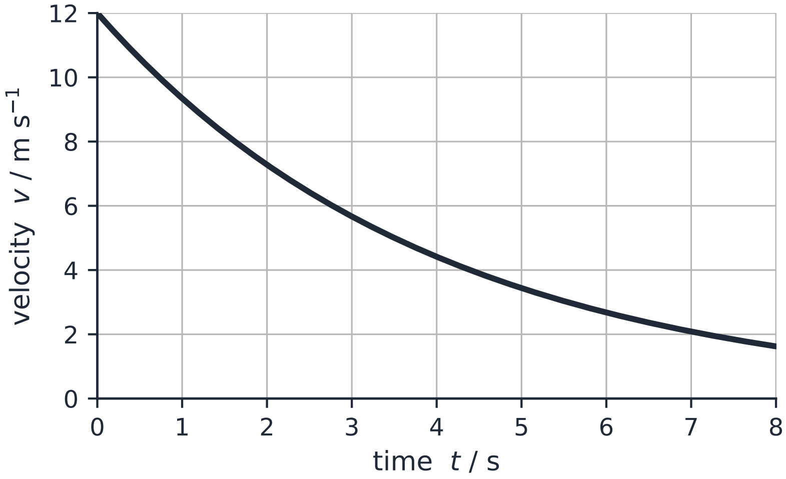


(a) Determine the change in velocity of the car between $t = 0$ and $t = 4.0$ s. [1]

(b) State the velocity of the car at $t = 6.0$ s. [1]

(c) **Sketch** the velocity–time graph of the car from $t = 0$ to $t = 8.0$ s. Label both axes and mark the values of velocity. [2]

2.4 A cyclist stops pedalling and slows down. The graph shows the velocity of the cyclist.

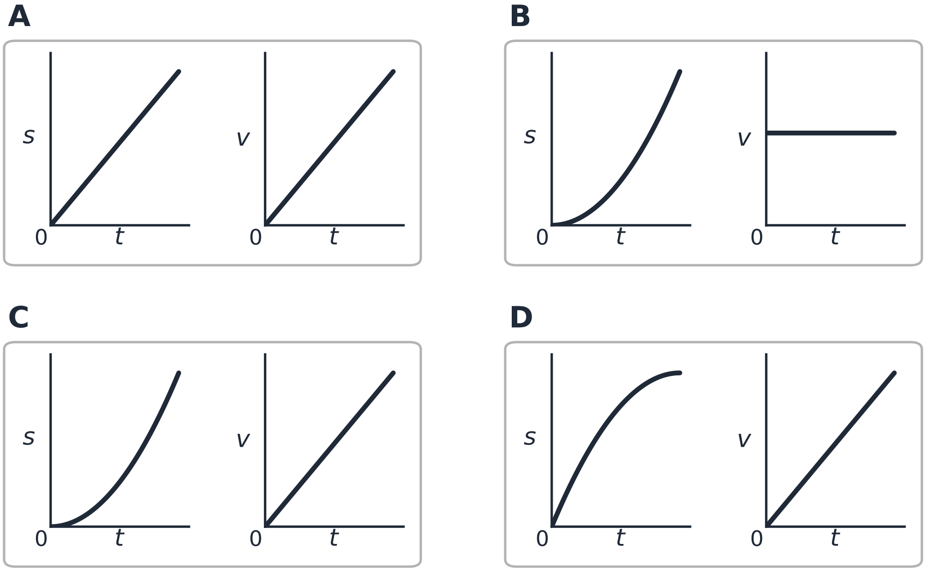


(a) By drawing a tangent, determine the acceleration of the cyclist at $t = 4.0$ s. [2]

(b) Estimate the distance travelled by the cyclist from $t = 0$ to $t = 8.0$ s by counting squares. [2]

3 Exam-style questions

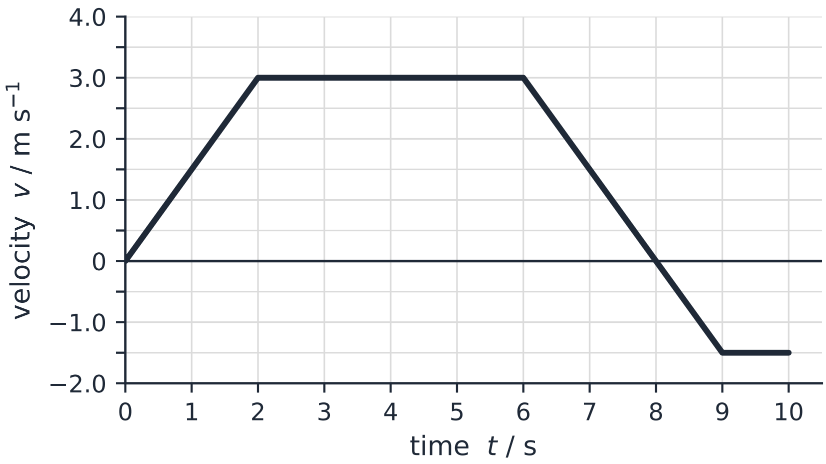
3.1 An object starts from rest and moves in a straight line. Which pair of graphs could show how its displacement s and its velocity v change with time t? [1]



3.2 The velocity–time graph of an object is a straight line from 8.0 m s^{-1} at $t = 0$ to -4.0 m s^{-1} at $t = 6.0$ s. What is the displacement of the object after 6.0 s? [1]

- **A** 12 m
- **B** 20 m
- **C** 24 m
- **D** 36 m

3.3 A remote-controlled toy car moves along a straight track. A **motion sensor** 运动传感器 records its velocity v. The graph shows how v varies with time t.



(a) **Define** acceleration. [1]

(b) **Compare** the acceleration of the car at $t = 1.0$ s with its acceleration at $t = 7.0$ s, in terms of magnitude and direction. [2]

(c) The car is at rest for an instant at $t = 8.0$ s. Determine the acceleration of the car at this instant. [2]

(d) Determine the greatest displacement of the car from its starting point, and the time at which it happens. [3]

(e) Determine the displacement of the car from its starting point at $t = 10.0$ s. [2]

(f) **Sketch** the variation with time of the displacement of the car from $t = 0$ to $t = 10.0$ s. Label both axes and mark the greatest displacement. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) the gradient is the **acceleration**

(b) the area is the **displacement**

2.2

(a) gradient of the first line

$$a = \frac{2.0\ \mathrm{m\ s^{-1}} - 0}{2.0\ \mathrm{s}} = \mathbf{1.0}\ \mathrm{m\ s^{-2}}$$

(b) $v = 0$ at $t = \mathbf{0}$, $\mathbf{8.0}$ s and $\mathbf{14}$ s

(c) going up, the area above the axis is a trapezium (or two triangles and a rectangle)

$$\tfrac{1}{2}(8.0\ \mathrm{s} + 4.0\ \mathrm{s})(2.0\ \mathrm{m\ s^{-1}}) = 12\ \mathrm{m}$$

- coming down, the area below the axis

$$\tfrac{1}{2}(6.0\ \mathrm{s} + 2.0\ \mathrm{s})(2.0\ \mathrm{m\ s^{-1}}) = 8.0\ \mathrm{m}$$

- total distance $= 12\ \mathrm{m} + 8.0\ \mathrm{m} = \mathbf{20}\ \mathrm{m}$

(d) displacement $= 12\ \mathrm{m} - 8.0\ \mathrm{m} = \mathbf{4.0}$ m above the starting point

2.3

(a) change in velocity = area under the acceleration–time graph

$$\Delta v = 2.0\ \mathrm{m\ s^{-2}} \times 4.0\ \mathrm{s} = \mathbf{8.0}\ \mathrm{m\ s^{-1}}$$

(b) $5.0\ \mathrm{m\ s^{-1}} + 8.0\ \mathrm{m\ s^{-1}} = 13\ \mathrm{m\ s^{-1}}$ at 4.0 s; the acceleration is then zero, so at 6.0 s the velocity is still **13** $\mathrm{m\ s^{-1}}$

(c)

- a straight line from $(0,\ 5.0\ \mathrm{m\ s^{-1}})$ to $(4.0\ \mathrm{s},\ 13\ \mathrm{m\ s^{-1}})$
- then a horizontal line at $13\ \mathrm{m\ s^{-1}}$ to 8.0 s, with the axes labelled $v\ /\ \mathrm{m\ s^{-1}}$ and $t\ /\ \mathrm{s}$

2.4

(a) tangent drawn at $t = 4.0$ s, for example through $(0,\ 8.8\ \mathrm{m\ s^{-1}})$ and $(8.0\ \mathrm{s},\ 0)$

$$a = \frac{0 - 8.8\ \mathrm{m\ s^{-1}}}{8.0\ \mathrm{s} - 0} = -\mathbf{1.1}\ \mathrm{m\ s^{-2}}$$

(accept -1.0 to -1.2)

(b) about 21 squares are at least half under the curve, and each is worth $1\ \mathrm{s} \times 2\ \mathrm{m\ s^{-1}} = 2\ \mathrm{m}$

$$\text{distance} \approx 21 \times 2\ \mathrm{m} = \mathbf{42}\ \mathrm{m}$$

(accept 40 m to 44 m)

3.1 C

- the curve in **C** gets steeper, so the velocity increases from zero, which matches a velocity line rising from the origin
- **A**: a straight displacement line means a constant velocity, not an increasing one
- **B**: a curve that gets steeper cannot have a constant velocity
- **D**: a curve that gets flatter means the velocity is decreasing

3.2 A

- the gradient is $-2.0\ \mathrm{m\ s^{-2}}$, so $v = 0$ at $t = 4.0$ s
- area above the axis and area below it

$$\tfrac{1}{2}(4.0\ \mathrm{s})(8.0\ \mathrm{m\ s^{-1}}) = 16\ \mathrm{m}, \qquad \tfrac{1}{2}(2.0\ \mathrm{s})(-4.0\ \mathrm{m\ s^{-1}}) = -4.0\ \mathrm{m}$$

- displacement = 16 m − 4.0 m = **12** m
- **B** is the distance; **C** treats the whole graph as one triangle; **D** ignores the minus sign

3.3

(a) acceleration is the **rate of change of velocity**

(b) the gradients are

$$\frac{3.0\ \mathrm{m\ s^{-1}}}{2.0\ \mathrm{s}} = 1.5\ \mathrm{m\ s^{-2}} \quad \text{and} \quad \frac{-1.5\ \mathrm{m\ s^{-1}} - 3.0\ \mathrm{m\ s^{-1}}}{9.0\ \mathrm{s} - 6.0\ \mathrm{s}} = -1.5\ \mathrm{m\ s^{-2}}$$

- magnitude: **the same**
- direction: **opposite**

(c) the acceleration is the gradient of the straight line from 6.0 s to 9.0 s, which includes 8.0 s

$$a = \frac{-1.5\ \mathrm{m\ s^{-1}} - 3.0\ \mathrm{m\ s^{-1}}}{9.0\ \mathrm{s} - 6.0\ \mathrm{s}} = \mathbf{-1.5}\ \mathrm{m\ s^{\mathbf{-2}}}$$

- it is not zero: the velocity is zero for an instant, but it is still changing

(d) the car moves forwards until $v = 0$, so the greatest displacement is at $t = \mathbf{8.0}$ s

- displacement = area under the line from 0 to 8.0 s

$$\tfrac{1}{2}(2.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + (4.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) + \tfrac{1}{2}(2.0\ \mathrm{s})(3.0\ \mathrm{m\ s^{-1}}) = \mathbf{18}\ \mathrm{m}$$

(e) area below the axis from 8.0 s to 10.0 s

$$\tfrac{1}{2}(1.0\ \mathrm{s})(-1.5\ \mathrm{m\ s^{-1}}) + (1.0\ \mathrm{s})(-1.5\ \mathrm{m\ s^{-1}}) = -2.25\ \mathrm{m}$$

- displacement = 18 m − 2.25 m = **16** m

(f)

- axes labelled s / m and t / s; a curve getting steeper to 3.0 m at 2.0 s, then a straight line to 15 m at 6.0 s
- a curve getting flatter to a maximum of 18 m at 8.0 s, flat at the top
- the displacement then decreasing, to about 16 m at 10.0 s