

2.1.1 Describing motion and displacement–time graphs

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • speed 速率 • distance 距离
• displacement 位移

Objective

Build the skills to answer exam questions on **describing motion** 描述运动: the exact definitions, average speed and average velocity, and reading a velocity from a **displacement–time graph** 位移–时间图像. This sheet covers syllabus 2.1, learning outcomes 1, 2 and 4.

You must be able to:

- **define** distance 距离, displacement 位移, speed 速率, velocity 速度 and acceleration 加速度
- tell average speed from average velocity, and change km h^{-1} into m s^{-1}
- add two displacements that are at right angles
- find a velocity from the gradient of a displacement–time graph, using a tangent for a curve
- **describe** a motion from the shape of its displacement–time graph

1 Worked examples

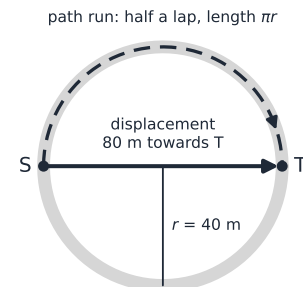
Study these first. Each one shows a **method** 方法 that a question later on this sheet needs.

■ Definitions the examiner accepts

quantity	definition	scalar 标量 or vector 矢量
distance	the total length of the path travelled	scalar
displacement	the distance in a straight line from the start, in a stated direction	vector
speed	the rate of change of distance	scalar
velocity	the rate of change of displacement	vector
acceleration	the rate of change of velocity	vector

■ Distance or displacement?

An athlete runs half a lap of a circular track of radius 40 m, from S to T, in 25 s.



- distance: the path is half the **circumference** 周长
- displacement: the straight line from S to T is the **diameter** 直径, 80 m towards T

$$\pi r = \pi \times 40 \text{ m} = 126 \text{ m}$$

- **average speed** 平均速率 = total distance ÷ total time

$$\frac{126 \text{ m}}{25 \text{ s}} = 5.0 \text{ m s}^{-1}$$

- **average velocity** 平均速度 = displacement ÷ total time

$$\frac{80 \text{ m}}{25 \text{ s}} = 3.2 \text{ m s}^{-1} \text{ towards T}$$

After a **whole** lap the displacement is zero, so the average velocity is zero. The average speed is not.

■ Adding two displacements

A walker goes 600 m due west and then 800 m due south. The two displacements are at right angles, so they are the two short sides of a right-angled triangle (Topic 1).

- size of the displacement, by Pythagoras

$$\sqrt{(600 \text{ m})^2 + (800 \text{ m})^2} = 1000 \text{ m}$$

- direction, as an angle measured from west towards south

$$\tan \theta = \frac{800 \text{ m}}{600 \text{ m}} \Rightarrow \theta = 53^\circ$$

- the displacement is 1000 m at 53° south of west; the distance walked is 1400 m

Units. To change km h^{-1} into m s^{-1} , multiply by 1000 (km to m) and divide by 3600 (h to s):

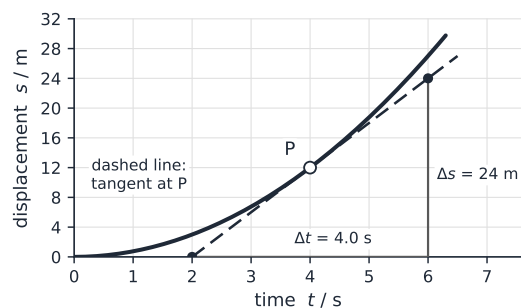
$$54 \text{ km h}^{-1} = \frac{54 \times 1000 \text{ m}}{3600 \text{ s}} = 15 \text{ m s}^{-1}$$

■ Velocity from a displacement–time graph

The **gradient** 斜率 of a displacement–time graph is the velocity. Use the shape to **describe** 描述 the motion:

- flat line: the object is at rest
- straight sloping line: constant velocity; a line sloping down means moving back towards the start (negative velocity)
- curve getting steeper: speeding up; curve getting flatter: slowing down

For a curve, draw a **tangent** 切线 at the time you need, make a **large** triangle on it, and find its gradient.



Example: the velocity of the car at P, where $t = 4.0 \text{ s}$.

- the tangent passes through (2.0 s, 0) and (6.0 s, 24 m)

$$v = \frac{\Delta s}{\Delta t} = \frac{24 \text{ m} - 0}{6.0 \text{ s} - 2.0 \text{ s}} = 6.0 \text{ m s}^{-1}$$

- a small triangle would make your reading errors much larger
- this is the **instantaneous** 瞬时 velocity at P; the average velocity over a time is the total displacement divided by that time

2 Practice

Now use the methods above.

2.1 The examiner often starts a question with a definition.

- (a) **Define** velocity. [1]

- (b) State one way in which velocity is different from speed. [1]

2.2 A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

- (a) State the total distance travelled. [1]

- (b) **Determine** the size of the cyclist's displacement from the start. (*Hint: the two displacements are at right angles.*) [1]

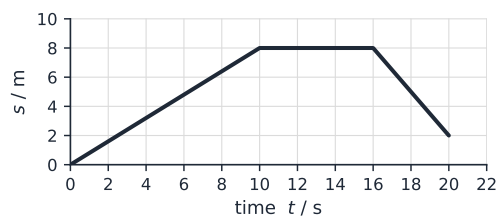
- (c) Determine the direction of the displacement, as an angle measured from east towards north. [1]

- (d) Calculate the average speed of the cyclist, in m s^{-1} . [2]

- (e) Calculate the size of the average velocity of the cyclist, in m s^{-1} . [1]

2.3 Change the units. (*Hint: 1 km = 1000 m and 1 h = 3600 s.*) (a) 72 km h^{-1} into m s^{-1} ; (b) 25 m s^{-1} into km h^{-1} . [2]

2.4 The displacement–time graph shows a model train on a straight track.

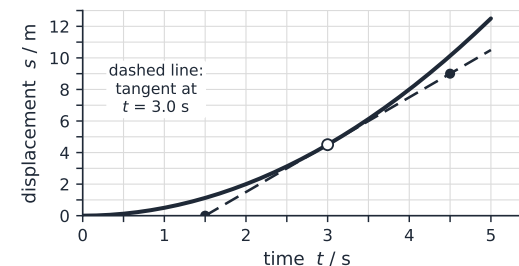


(a) Determine the velocity of the train between $t = 0$ and $t = 10 \text{ s}$. [1]

(b) Describe the motion of the train between $t = 10 \text{ s}$ and $t = 16 \text{ s}$. [1]

(c) Determine the velocity of the train between $t = 16 \text{ s}$ and $t = 20 \text{ s}$. [1]

2.5 A trolley 小车 is released from rest at the top of a ramp 斜坡. The graph shows its displacement down the ramp. A tangent has been drawn at $t = 3.0 \text{ s}$.



(a) Use the tangent to determine the velocity of the trolley at $t = 3.0 \text{ s}$. [2]

(b) State how the graph shows that the trolley is speeding up. [1]

3 Exam-style questions

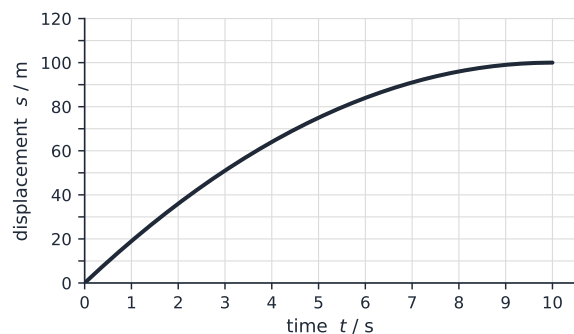
3.1 A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s . What are her average speed and the size of her average velocity? [1]

- A speed 3.3 m s^{-1} , velocity 0
- B speed 3.3 m s^{-1} , velocity 0.42 m s^{-1}
- C speed 0.42 m s^{-1} , velocity 3.3 m s^{-1}
- D speed 3.3 m s^{-1} , velocity 3.3 m s^{-1}

3.2 A bus travels 12 km at an average speed of 40 km h^{-1} , then another 12 km at 60 km h^{-1} . What is its average speed for the whole journey? [1]

- A 13 m s^{-1}
- B 14 m s^{-1}
- C 48 m s^{-1}
- D 50 m s^{-1}

3.3 The graph shows the displacement s of a car along a straight road as the driver brakes to a stop.



- (a) **Describe** the motion of the car from $t = 0$ to $t = 10$ s. [2]

- (b) By drawing a tangent, determine the velocity of the car at $t = 4.0$ s. [3]

- (c) Determine the average velocity of the car from $t = 0$ to $t = 10$ s. [2]

- (d) Explain why your answers to (b) and (c) are different. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- (a) velocity is the **rate of change of displacement**
 (b) velocity is a vector, so it has a direction; speed is a scalar, so it has none

2.2

- (a) $3.0 \text{ km} + 5.0 \text{ km} = \mathbf{8.0 \text{ km}}$
 (b) the displacements are at right angles, so use Pythagoras

$$\sqrt{(3.0 \text{ km})^2 + (5.0 \text{ km})^2} = \mathbf{5.8 \text{ km}}$$

- (c) opposite side north, adjacent side east

$$\tan \theta = \frac{5.0 \text{ km}}{3.0 \text{ km}} \Rightarrow \theta = \mathbf{59^\circ}$$

north of east

- (d) time = $25 \times 60 \text{ s} = 1500 \text{ s}$ and distance = 8000 m

$$\text{average speed} = \frac{8000 \text{ m}}{1500 \text{ s}} = \mathbf{5.3 \text{ m s}^{-1}}$$

- (e) use the displacement, not the distance

$$\text{average velocity} = \frac{5830 \text{ m}}{1500 \text{ s}} = \mathbf{3.9 \text{ m s}^{-1}}$$

2.3

- (a) multiply by 1000 and divide by 3600

$$72 \text{ km h}^{-1} = \frac{72 \times 1000 \text{ m}}{3600 \text{ s}} = \mathbf{20 \text{ m s}^{-1}}$$

- (b) in one hour the car travels $25 \text{ m} \times 3600 = 90\,000 \text{ m} = 90 \text{ km}$, so the speed is $\mathbf{90 \text{ km h}^{-1}}$

2.4

- (a) velocity = gradient

$$v = \frac{8.0 \text{ m} - 0}{10 \text{ s} - 0} = \mathbf{0.80 \text{ m s}^{-1}}$$

- (b) the line is flat, so the displacement does not change: the train is **at rest**

- (c) gradient of the line from 16 s to 20 s

$$v = \frac{2.0 \text{ m} - 8.0 \text{ m}}{20 \text{ s} - 16 \text{ s}} = \mathbf{-1.5 \text{ m s}^{-1}}$$

- the minus sign means the train moves back towards its start

2.5

- (a) the tangent passes through (1.5 s, 0) and (4.5 s, 9.0 m)

$$v = \frac{9.0 \text{ m} - 0}{4.5 \text{ s} - 1.5 \text{ s}} = \mathbf{3.0 \text{ m s}^{-1}}$$

- (b) the curve gets **steeper**: its gradient, the velocity, increases with time

3.1 B

- distance = $2.5 \times 400 \text{ m} = 1000 \text{ m}$

$$\text{average speed} = \frac{1000 \text{ m}}{300 \text{ s}} = 3.3 \text{ m s}^{-1}$$

- after 2.5 laps the runner is half a lap from the start, so the displacement is one diameter

$$d = \frac{400 \text{ m}}{\pi} = 127 \text{ m}, \quad \text{average velocity} = \frac{127 \text{ m}}{300 \text{ s}} = \mathbf{0.42 \text{ m s}^{-1}}$$

- A** is true only after whole laps; **C** swaps them; **D** uses the distance twice

3.2 A

- time for each part

$$t_1 = \frac{12 \text{ km}}{40 \text{ km h}^{-1}} = 0.30 \text{ h}, \quad t_2 = \frac{12 \text{ km}}{60 \text{ km h}^{-1}} = 0.20 \text{ h}$$

- average speed = total distance $\div$ total time

$$\frac{24 \text{ km}}{0.50 \text{ h}} = 48 \text{ km h}^{-1} = \frac{48 \times 1000 \text{ m}}{3600 \text{ s}} = \mathbf{13 \text{ m s}^{-1}}$$

- B** averages the two speeds, but the bus spends longer at the lower speed; **C** and **D** forget to change the units

3.3

- (a)

- the displacement increases, so the car moves forwards
- the gradient decreases to zero, so the car slows down and stops at $t = 10 \text{ s}$

- (b)

- tangent drawn at $t = 4.0 \text{ s}$
- gradient from a large triangle, for example through (0, 16 m) and (8.0 s, 112 m)

$$v = \frac{112 \text{ m} - 16 \text{ m}}{8.0 \text{ s} - 0} = \mathbf{12 \text{ m s}^{-1}}$$

(accept 11 to 13)

- (c) average velocity = displacement $\div$ time

$$\frac{100 \text{ m}}{10 \text{ s}} = \mathbf{10 \text{ m s}^{-1}}$$

- (d) (b) is the velocity at one instant; (c) is the total displacement divided by the total time