2.1.1 Describing motion and displacement–time graphs

Name: ______________ Class: __________ Date: ____________ **Total: 26 marks**

KEY WORDS velocity 速度 • acceleration 加速度 • speed 速率 • distance 距离 • displacement 位移

Objective

Build the skills to answer exam questions on **describing motion** 描述运动: the exact definitions, average speed and average velocity, and reading a velocity from a **displacement–time graph** 位移 – 时间图像. This sheet covers syllabus 2.1, learning outcomes 1, 2 and 4.

You must be able to:

- **define** distance 距离, displacement 位移, speed 速率, velocity 速度 and acceleration 加速度
- tell average speed from average velocity, and change km h^{-1} into m s^{-1}
- add two displacements that are at right angles
- find a velocity from the gradient of a displacement–time graph, using a tangent for a curve
- **describe** a motion from the shape of its displacement–time graph

1 Worked examples

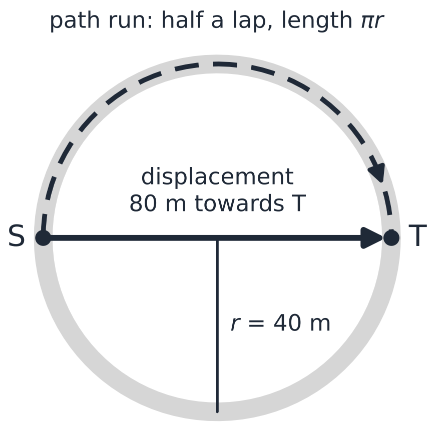
Study these first. Each one shows a **method** 方法 that a question later on this sheet needs.

■ Definitions the examiner accepts

quantity	**definition**	**scalar 标量 or vector 矢量**
distance	the total length of the path travelled	scalar
displacement	the distance in a straight line from the start, in a stated direction	vector
speed	the rate of change of distance	scalar
velocity	the rate of change of displacement	vector
acceleration	the rate of change of velocity	vector

Distance or displacement?

An athlete runs half a lap of a circular track of radius 40 m, from S to T, in 25 s.



- distance: the path is half the **circumference** 周长

$$\pi r = \pi \times 40 \text{ m} = 126 \text{ m}$$

- displacement: the straight line from S to T is the **diameter** 直径, 80 m towards T
- **average speed** 平均速率 = total distance ÷ total time

$$\frac{126 \text{ m}}{25 \text{ s}} = 5.0 \text{ m s}^{-1}$$

- **average velocity** 平均速度 = displacement ÷ total time

$$\frac{80 \text{ m}}{25 \text{ s}} = 3.2 \text{ m s}^{-1} \text{ towards T}$$

After a **whole** lap the displacement is zero, so the average velocity is zero. The average speed is not.

Adding two displacements

A walker goes 600 m due west and then 800 m due south. The two displacements are at right angles, so they are the two short sides of a right-angled triangle (Topic 1).

- size of the displacement, by Pythagoras

$$\sqrt{(600 \text{ m})^2 + (800 \text{ m})^2} = 1000 \text{ m}$$

- direction, as an angle measured from west towards south

$$\tan\theta = \frac{800 \text{ m}}{600 \text{ m}} \Rightarrow \theta = 53^\circ$$

- the displacement is 1000 m at 53° south of west; the distance walked is 1400 m

Units. To change km h^{-1} into m s^{-1}, multiply by 1000 (km to m) and divide by 3600 (h to s):

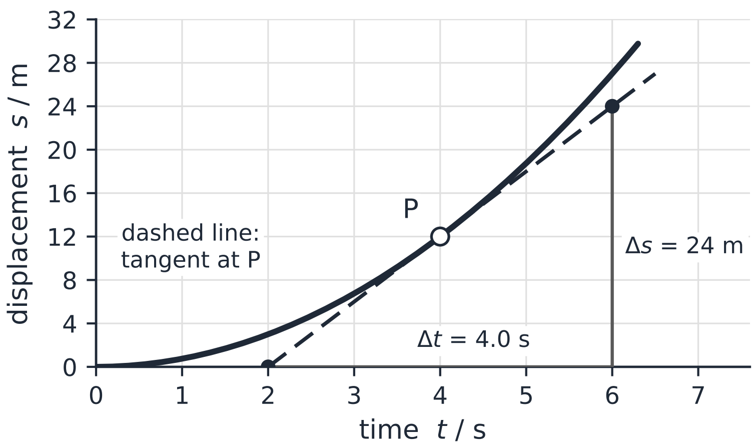
$$54 \text{ km h}^{-1} = \frac{54 \times 1000 \text{ m}}{3600 \text{ s}} = 15 \text{ m s}^{-1}$$

■ Velocity from a displacement–time graph

The **gradient** 斜率 of a displacement–time graph is the velocity. Use the shape to **describe** 描述 the motion:

- flat line: the object is at rest
- straight sloping line: constant velocity; a line sloping down means moving back towards the start (negative velocity)
- curve getting steeper: speeding up; curve getting flatter: slowing down

For a curve, draw a **tangent** 切线 at the time you need, make a **large** triangle on it, and find its gradient.



Example: the velocity of the car at P, where $t = 4.0$ s.

- the tangent passes through (2.0 s, 0) and (6.0 s, 24 m)

$$v = \frac{\Delta s}{\Delta t} = \frac{24 \text{ m} - 0}{6.0 \text{ s} - 2.0 \text{ s}} = 6.0 \text{ m s}^{-1}$$

- a small triangle would make your reading errors much larger
- this is the **instantaneous** 瞬时 velocity at P; the average velocity over a time is the total displacement divided by that time

2 Practice

Now use the methods above.

2.1 The examiner often starts a question with a definition.

(a) **Define** velocity. [1]

(b) State one way in which velocity is different from speed. [1]

2.2 A cyclist rides 3.0 km due east and then 5.0 km due north. The whole ride takes 25 minutes.

(a) State the total distance travelled. [1]

(b) **Determine** the size of the cyclist's displacement from the start. *(Hint: the two displacements are at right angles.)* [1]

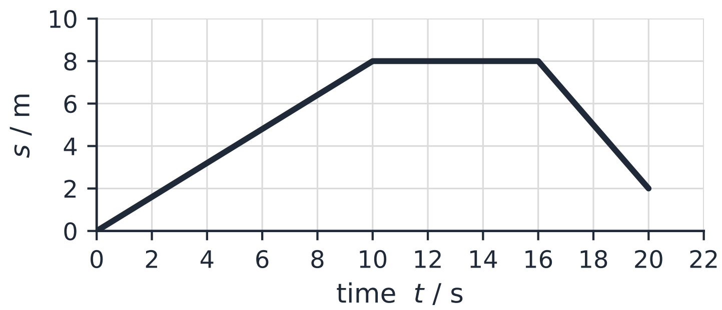
(c) Determine the direction of the displacement, as an angle measured from east towards north. [1]

(d) Calculate the average speed of the cyclist, in m s^{-1}. [2]

(e) Calculate the size of the average velocity of the cyclist, in m s^{-1}. [1]

2.3 Change the units. *(Hint: 1 km = 1000 m and 1 h = 3600 s.)* (a) 72 km h^{-1} into m s^{-1}; (b) 25 m s^{-1} into km h^{-1}. [2]

2.4 The displacement–time graph shows a model train on a straight track.

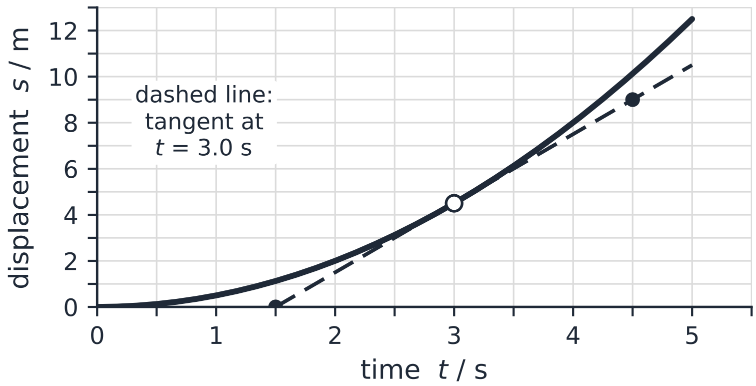


(a) Determine the velocity of the train between $t = 0$ and $t = 10$ s. [1]

(b) Describe the motion of the train between $t = 10$ s and $t = 16$ s. [1]

(c) Determine the velocity of the train between $t = 16$ s and $t = 20$ s. [1]

2.5 A **trolley** 小车 is released from rest at the top of a **ramp** 斜坡. The graph shows its displacement down the ramp. A tangent has been drawn at $t = 3.0$ s.



(a) Use the tangent to determine the velocity of the trolley at $t = 3.0$ s. [2]

(b) State how the graph shows that the trolley is speeding up. [1]

3 Exam-style questions

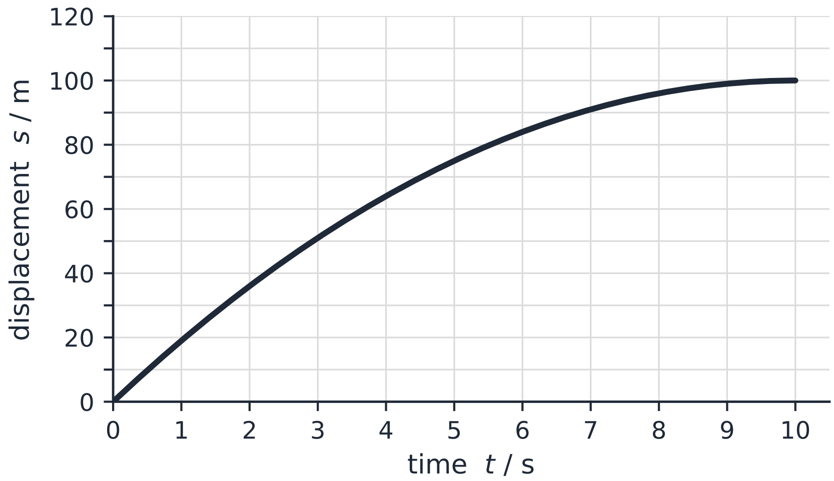
3.1 A runner completes 2.5 laps of a circular track of circumference 400 m in 300 s. What are her average speed and the size of her average velocity? [1]

- **A** speed 3.3 m s^{-1}, velocity 0
- **B** speed 3.3 m s^{-1}, velocity 0.42 m s^{-1}
- **C** speed 0.42 m s^{-1}, velocity 3.3 m s^{-1}
- **D** speed 3.3 m s^{-1}, velocity 3.3 m s^{-1}

3.2 A bus travels 12 km at an average speed of 40 km h^{-1}, then another 12 km at 60 km h^{-1}. What is its average speed for the whole journey? [1]

- **A** 13 m s^{-1}
- **B** 14 m s^{-1}
- **C** 48 m s^{-1}
- **D** 50 m s^{-1}

3.3 The graph shows the displacement s of a car along a straight road as the driver brakes to a stop.



(a) **Describe** the motion of the car from $t = 0$ to $t = 10$ s. [2]

(b) By drawing a tangent, determine the velocity of the car at $t = 4.0$ s. [3]

(c) Determine the average velocity of the car from $t = 0$ to $t = 10$ s. [2]

(d) Explain why your answers to (b) and (c) are different. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) velocity is the **rate of change of displacement**

(b) velocity is a vector, so it has a direction; speed is a scalar, so it has none

2.2

(a) $3.0\ \text{km} + 5.0\ \text{km} = \mathbf{8.0}\ \text{km}$

(b) the displacements are at right angles, so use Pythagoras

$$\sqrt{(3.0\ \text{km})^2 + (5.0\ \text{km})^2} = \mathbf{5.8}\ \text{km}$$

(c) opposite side north, adjacent side east

$$\tan\theta = \frac{5.0\ \text{km}}{3.0\ \text{km}} \Rightarrow \theta = \mathbf{59}^\circ$$

north of east

(d) time $= 25 \times 60\ \text{s} = 1500\ \text{s}$ and distance $= 8000\ \text{m}$

$$\text{average speed} = \frac{8000\ \text{m}}{1500\ \text{s}} = \mathbf{5.3}\ \text{m s}^{-1}$$

(e) use the displacement, not the distance

$$\text{average velocity} = \frac{5830\ \text{m}}{1500\ \text{s}} = \mathbf{3.9}\ \text{m s}^{-1}$$

2.3

(a) multiply by 1000 and divide by 3600

$$72\ \text{km h}^{-1} = \frac{72 \times 1000\ \text{m}}{3600\ \text{s}} = \mathbf{20}\ \text{m s}^{-1}$$

(b) in one hour the car travels $25\ \text{m} \times 3600 = 90\,000\ \text{m} = 90\ \text{km}$, so the speed is $\mathbf{90}\ \text{km h}^{-1}$

2.4

(a) velocity = gradient

$$v = \frac{8.0\ \text{m} - 0}{10\ \text{s} - 0} = \mathbf{0.80}\ \text{m s}^{-1}$$

(b) the line is flat, so the displacement does not change: the train is **at rest**

(c) gradient of the line from 16 s to 20 s

$$v = \frac{2.0\ \text{m} - 8.0\ \text{m}}{20\ \text{s} - 16\ \text{s}} = \mathbf{-1.5}\ \text{m s}^{-1}$$

- the minus sign means the train moves back towards its start

2.5

(a) the tangent passes through (1.5 s, 0) and (4.5 s, 9.0 m)

$$v = \frac{9.0\ \text{m} - 0}{4.5\ \text{s} - 1.5\ \text{s}} = \mathbf{3.0}\ \text{m s}^{-1}$$

(b) the curve gets **steeper**: its gradient, the velocity, increases with time

3.1 B

- distance = 2.5 × 400 m = 1000 m

$$\text{average speed} = \frac{1000\ \text{m}}{300\ \text{s}} = 3.3\ \text{m s}^{-1}$$

- after 2.5 laps the runner is half a lap from the start, so the displacement is one diameter

$$d = \frac{400\ \text{m}}{\pi} = 127\ \text{m}, \qquad \text{average velocity} = \frac{127\ \text{m}}{300\ \text{s}} = \mathbf{0.42}\ \text{m s}^{-1}$$

- **A** is true only after whole laps; **C** swaps them; **D** uses the distance twice

3.2 A

- time for each part

$$t_1 = \frac{12\ \text{km}}{40\ \text{km h}^{-1}} = 0.30\ \text{h}, \qquad t_2 = \frac{12\ \text{km}}{60\ \text{km h}^{-1}} = 0.20\ \text{h}$$

- average speed = total distance ÷ total time

$$\frac{24\ \text{km}}{0.50\ \text{h}} = 48\ \text{km h}^{-1} = \frac{48 \times 1000\ \text{m}}{3600\ \text{s}} = \mathbf{13}\ \text{m s}^{-1}$$

- **B** averages the two speeds, but the bus spends longer at the lower speed; **C** and **D** forget to change the units

3.3

(a)

- the displacement increases, so the car moves forwards
- the gradient decreases to zero, so the car slows down and stops at $t = 10$ s

(b)

- tangent drawn at $t = 4.0$ s
- gradient from a large triangle, for example through (0, 16 m) and (8.0 s, 112 m)

$$v = \frac{112\ \text{m} - 16\ \text{m}}{8.0\ \text{s} - 0} = \mathbf{12}\ \text{m s}^{-1}$$

(accept 11 to 13)

(c) average velocity = displacement ÷ time

$$\frac{100\ \text{m}}{10\ \text{s}} = \mathbf{10}\ \text{m s}^{\mathbf{-1}}$$

(d) (b) is the velocity at one instant; (c) is the total displacement divided by the total time