A-LEVEL PHYSICS • EXERCISE SHEET

19.3 Discharging a capacitor

Name: ____________ Class: ________ Date: __________ **Total: 29 marks**

KEY WORDS time constant 时间常数 • exponential decay 指数衰减 • resistor 电阻器 • resistance 电阻 • capacitor 电容器

Objective

Build the skills to answer exam questions on a **capacitor discharging** through a resistor.

You must be able to:

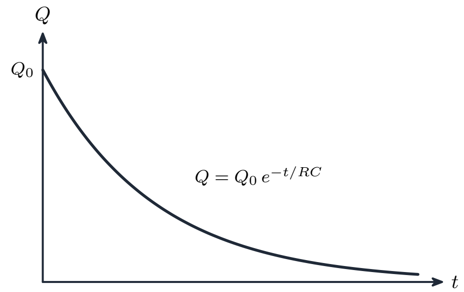
- use the **time constant** $\tau = RC$
- use $x = x_0 e^{-t/RC}$ for charge, p.d. or current
- interpret the discharge graphs

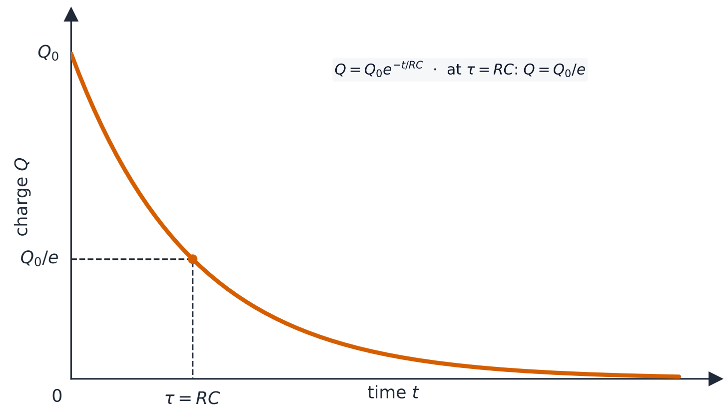
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Exponential decay

When a charged capacitor discharges through a resistor, the charge (and V, I) decays **exponentially**:





Exponential discharge of a capacitor

$$Q = Q_0 e^{-t/RC}, \quad V = V_0 e^{-t/RC}, \quad I = I_0 e^{-t/RC}.$$

The **time constant** $\tau = RC$ is the time for the charge to fall to $\frac{1}{e} \approx 37\%$ of its initial value.

Worked example. 470 μF through 2.0 kΩ: $\tau = RC = (2000)(470 \times 10^{-6}) = 0.94$ s.

■ Finding the time — take logs

To find the **time** for the charge (or V, or I) to fall to a given value, take the **natural log** of both sides of $V = V_0 e^{-t/RC}$:

$$\ln\frac{V}{V_0} = -\frac{t}{RC} \quad \Longrightarrow \quad t = -RC\,\ln\frac{V}{V_0}.$$

Worked example. For the capacitor above (charged to 6.0 V, $\tau = 0.94$ s), the time for the p.d. to fall to 2.0 V is $t = -0.94 \ln\frac{2.0}{6.0} = -0.94 \times (-1.10) = 1.0$ s.

2 Practice

Now apply the methods above.

2.1 Write the formula for the **time constant**. [1]

2.2 A 100 μF capacitor discharges through a 10 kΩ resistor. Find the **time constant**. [2]

2.3 State what happens to the **charge** as a capacitor discharges. [1]

2.4 Write the discharge equation for **charge**. [1]

3 Exam-style questions

3.1 The time constant RC is the time for the charge to fall to: [1]

- **A** zero
- **B** half its initial value
- **C** $1/e$ (about 37%) of its initial value
- **D** $1/2e$ of its initial value

3.2 A 470 μF capacitor charged to 6.0 V discharges through a 2.0 kΩ resistor. Calculate (a) the time constant, (b) the p.d. after 1.0 s. [4]

3.3 Sketch a graph of **charge against time** for a discharging capacitor. [2]

3.4 State what happens to the time constant if the **resistance** is doubled. [1]

3.5 A 220 μF capacitor charged to 9.0 V discharges through a 15 kΩ resistor. Calculate the **time** for the p.d. to fall to 3.0 V. [4]

3.6 A capacitor C is charged and then discharged through a resistor R. A data logger records the p.d. V_R across the resistor and the p.d. V_C across the capacitor. At $t = 0$ the p.d. across the capacitor is 9.0 V, and the initial current is 1.8 mA. The p.d. across the capacitor falls to 3.3 V after 12 s.

(a) **State** the relationship between V_C and V_R during the discharge, and give a reason. [2]

(b) **Determine** the resistance R. [2]

(c) **Determine** the time constant τ of the circuit. [3]

(d) Hence **determine** the capacitance C, in μF. [2]

(e) **Explain**, in terms of the current in the circuit, why the decay of the charge on the capacitor is **exponential**. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\tau = RC$

2.2

- $\tau = RC$

$$\tau = (10 \times 10^3\ \Omega)(100 \times 10^{-6}\ \text{F}) = \mathbf{1.0}\ \text{s}$$

2.3

- it **decreases exponentially** towards zero

2.4

- $Q = Q_0 e^{-t/RC}$

3.1 C

- after one time constant, $Q = Q_0/e$ (about 37%)

3.2

(a) $\tau = RC$

$$\tau = (2.0 \times 10^3\ \Omega)(470 \times 10^{-6}\ \text{F}) = \mathbf{0.94}\ \text{s}$$

(b) $V = V_0 e^{-t/RC}$

$$V = (6.0\ \text{V})\, e^{-1.0/0.94} = (6.0\ \text{V})(0.345) = \mathbf{2.1}\ \text{V}$$

3.3

- a curve starting at the initial charge and **decaying exponentially** towards zero
- with the same shape (never quite reaching zero)

3.4

- it **doubles** ($\tau = RC$)

3.5

- $\tau = RC$

$$\tau = (15 \times 10^3\ \Omega)(220 \times 10^{-6}\ \text{F}) = 3.3\ \text{s}$$

- $t = -RC \ln(V/V_0)$

$$t = -3.3\ \text{s} \times \ln \frac{3.0\ \text{V}}{9.0\ \text{V}} = -3.3\ \text{s} \times (-1.10) = \mathbf{3.6}\ \text{s}$$

3.6

(a) they are **equal** at every instant, $V_C = V_R$

- the capacitor and resistor form a single loop with no other source, so the p.d. driving the current is the capacitor's own p.d.

(b) $R = V/I$

$$R = \frac{9.0\ \text{V}}{1.8 \times 10^{-3}\ \text{A}} = \mathbf{5.0 \times 10^3}$$

(c) $V = V_0 e^{-t/\tau}$, so $3.3/9.0 = e^{-12/\tau}$

$$\ln(0.367) = -\frac{12\ \text{s}}{\tau} \Rightarrow \tau = \mathbf{12}\ \text{s}$$

- the p.d. has fallen to $1/e$ of its start, which is what the time constant means

(d) $\tau = RC$, so $C = \tau/R$

$$C = \frac{12\ \text{s}}{5.0 \times 10^3\ \Omega} = 2.4 \times 10^{-3}\ \text{F} = \mathbf{2.4 \times 10^3}\ \mu\text{F}$$

(e) the current is $I = V_C/R$, and $V_C \propto Q$, so the current — the rate at which charge leaves — is **proportional to the charge still on the capacitor**

- a quantity whose rate of decrease is proportional to itself decays exponentially
- so as the charge falls the current falls with it, the discharge slows, and Q never quite reaches zero