A-LEVEL PHYSICS • EXERCISE SHEET

19.2 Energy stored in a capacitor

Name: ____________ Class: ________ Date: __________ **Total: 23 marks**

KEY WORDS energy 能量 • capacitor 电容器

Objective

Build the skills to answer exam questions on the **energy stored in a capacitor**.

You must be able to:

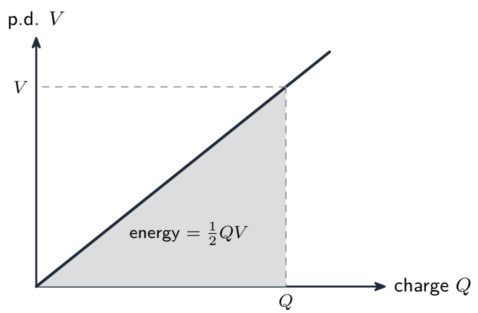
- find energy from the **area under the potential–charge graph**
- use $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$

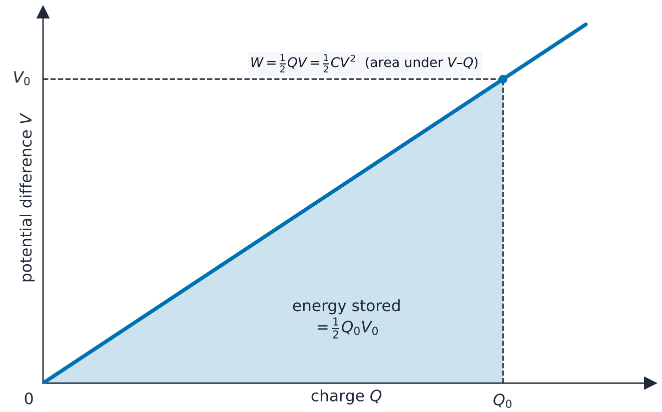
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Energy stored

The energy is the **area under the V–Q graph** — a triangle:





Energy stored is the area under the Q-V graph

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

The factor $\frac{1}{2}$ appears because the **average** p.d. during charging is $V/2$ (it grows from 0 to V).

Worked example. 100 μF at 12 V: $W = \frac{1}{2}CV^2 = \frac{1}{2}(100\times10^{-6})(12)^2 = 7.2\times10^{-3}$ J.

2 Practice

Now apply the methods above.

2.1 Write the **three** forms of the capacitor energy formula. [1]

2.2 Find the energy stored in a 100 μF capacitor charged to 12 V. [2]

2.3 State what the **area** under a potential–charge graph represents. [1]

3 Exam-style questions

3.1 The energy stored in a capacitor is: [1]

- **A** QV
- **B** $\frac{1}{2}QV$
- **C** CV
- **D** Q^2C

3.2 A 220 μF capacitor stores 4.0 mJ of energy. Calculate the **voltage** across it. [3]

3.3 Explain why the energy stored is $\frac{1}{2}QV$ and **not** QV. [2]

3.4 A defibrillator stores energy in a 32 μF capacitor charged to 2.5 kV, and delivers it to a patient in a pulse lasting 8.0 ms.

(a) **Determine** the energy stored by the capacitor when fully charged. [2]

(b) **Determine** the charge stored. [2]

(c) In the pulse, 75% of the stored energy is delivered to the patient. **Determine** the average power delivered. [3]

(d) The capacitor is charged instead to 1.8 kV. **Determine** the percentage by which the stored energy falls, and **explain** why it is not the same percentage as the fall in p.d. [3]

(e) On the same axes, **sketch** how the energy stored varies with the p.d. across the capacitor, and how the **charge** stored varies with it. **Label** each line. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/(2C)$

2.2

- $W = \frac{1}{2}CV^2$

$$W = \tfrac{1}{2}(100 \times 10^{-6}\ \text{F})(12\ \text{V})^2 = \mathbf{7.2 \times 10^{-3}}\ \text{J}$$

2.3

- the **energy stored** in the capacitor

3.1 B

- $W = \frac{1}{2}QV$

3.2

- $W = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{2W/C}$

$$V = \sqrt{\frac{2 \times 4.0 \times 10^{-3}\ \text{J}}{220 \times 10^{-6}\ \text{F}}} = \mathbf{6.0}\ \text{V}$$

3.3

- as the capacitor charges, the p.d. rises from 0 to V
- the **average** p.d. is $V/2$, so the work done is $Q \times (V/2) = \frac{1}{2}QV$, not QV

3.4

(a) $E = \frac{1}{2}CV^2$

$$E = \tfrac{1}{2}(32 \times 10^{-6}\ \text{F})(2500\ \text{V})^2 = \mathbf{100}\ \text{J}$$

(b) $Q = CV$

$$Q = (32 \times 10^{-6}\ \text{F})(2500\ \text{V}) = \mathbf{8.0 \times 10^{-2}}\ \text{C}$$

(c) energy delivered $= 0.75 \times 100\ \text{J} = 75\ \text{J}$

$$P = \frac{E}{t} = \frac{75\ \text{J}}{8.0 \times 10^{-3}\ \text{s}} = \mathbf{9.4 \times 10^3}\ \text{W}$$

(d) new energy $= \frac{1}{2}(32 \times 10^{-6}\ \text{F})(1800\ \text{V})^2 = 51.8\ \text{J}$

$$\frac{100\ \text{J} - 51.8\ \text{J}}{100\ \text{J}} = \mathbf{48}\%$$

- the p.d. fell by only 28%, because $E \propto V^2$, so the fractional change in energy is roughly **twice** the fractional change in p.d.

(e) charge: a **straight line through the origin**, since $Q = CV$

- energy: a **curve through the origin** getting steeper, since $E \propto V^2$
- both labelled, with axes labelled