A-LEVEL PHYSICS • EXERCISE SHEET

# 17.2 Energy in simple harmonic motion

Name: ____________ Class: _________ Date: __________ **Total: 29 marks**

**KEY WORDS** kinetic energy 动能 • damping 阻尼 • displacement 位移 • equilibrium 平衡 • amplitude 振幅

## Objective

Build the skills to answer exam questions on **energy in simple harmonic motion**.

**You must be able to:**

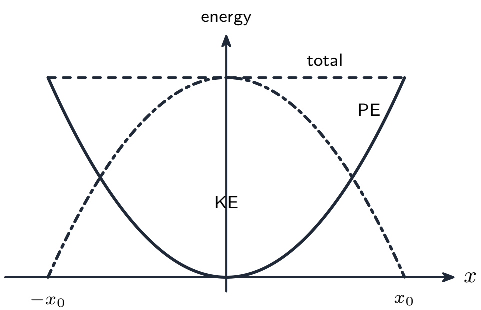
- describe the **KE** ↔ **PE** interchange during SHM
- use $E = \frac{1}{2}m\omega^2 x_0^2$ for the total energy

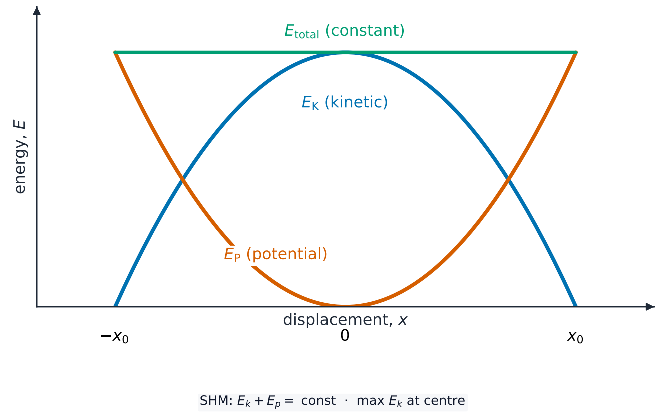
## 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ Energy interchange

KE and PE swap as the oscillator moves; with no damping the **total energy is constant**:




*Energy interchange in simple harmonic motion*

- at $x = 0$: KE is **maximum**, PE minimum;
- at $x = \pm x_0$: KE $= 0$, PE **maximum**.

### ■ Total energy

$$E = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m\omega^2 x_0^2.$$

So $E \propto x_0^2$ (doubling the amplitude gives **four times** the energy) and $E \propto \omega^2$.

### ■ Energy varies at twice the frequency

In one full oscillation the object passes through the centre (KE maximum) **twice**, so the **KE and PE vary at twice** the frequency of the displacement. On a graph of KE against **time**, one repeat of the KE pattern takes **half** the oscillation period — so if the KE graph repeats every $t_E$, the oscillation period is $T = 2t_E$.

## 2 Practice

Now apply the methods above.

**2.1** Describe the **energy interchange** during SHM. [2]

**2.2** State where the **kinetic energy** is maximum and where the **potential energy** is maximum. [2]

**2.3** Write the formula for the **total energy** of a simple harmonic oscillator. [1]

## 3 Exam-style questions

**3.1** In SHM, the total energy is proportional to: [1]

- **A** the amplitude $x_0$
- **B** the amplitude squared $x_0^2$
- **C** $1/x_0$
- **D** the period $T$

**3.2** A 0.20 kg mass oscillates with $\omega = 10$ rad s$^{-1}$ and amplitude 0.050 m. Calculate the **total energy**. [3]

**3.3** Sketch how the **kinetic energy** and **potential energy** vary with displacement in SHM. [2]

**3.4** State what happens to the total energy if the **amplitude is doubled**. [1]



**3.5** A graph of the **kinetic energy** of an oscillator against **time** repeats every 0.40 s.

**(a)** State the **period** of the oscillation, explaining your answer. [2]

**(b)** Calculate the **angular frequency** $\omega$ of the oscillation. [2]

**3.6** A mass hangs on a spring and oscillates vertically with simple harmonic motion. A data logger records its **kinetic energy** against time, giving a curve that repeats every 0.40 s. The mass is 0.25 kg and the amplitude of the oscillation is 4.0 cm.

(a) **Determine** the period of the oscillation of the **mass**, and explain why it is not 0.40 s. [2]

(b) **Determine** the angular frequency of the oscillation. [2]

(c) **Determine** the total energy of the oscillation. [3]

(d) **Determine** the speed of the mass when its displacement is 2.0 cm. [3]

(e) The oscillation is **lightly damped**. **State and explain** what happens to the time between successive maxima of the kinetic-energy curve, and to the height of those maxima. [3]



## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- energy is continually transferred between **kinetic** and **potential** forms
- with no damping the **total** stays constant

**2.2**

- KE is maximum at the **equilibrium** ($x = 0$)
- PE is maximum at the **extremes** ($x = \pm x_0$)

**2.3**

- $E = \frac{1}{2}m\omega^2 x_0^2$

**3.1 B**

- $E \propto x_0^2$, so energy is proportional to the **amplitude squared**

**3.2**

- $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \tfrac{1}{2}(0.20\ \text{kg})(10\ \text{rad s}^{-1})^2(0.050\ \text{m})^2 = \mathbf{0.025}\ \text{J}$$

**3.3**

- KE: a **downward** parabola, maximum at $x = 0$, zero at $\pm x_0$
- PE: an **upward** parabola, zero at $x = 0$, maximum at $\pm x_0$; their sum is constant

**3.4**

- it becomes **four times** larger ($E \propto x_0^2$)

**3.5**

(a) the KE varies at **twice** the oscillation frequency (the object passes the centre twice per cycle)

- so the oscillation period is **twice** the KE-graph period: $T = 2 \times 0.40\ \text{s} = \mathbf{0.80}\ \text{s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\ \text{s}} = \mathbf{7.9}\ \text{rad s}^{-1}$$

**3.6**

(a) the kinetic energy is zero at **both** ends of the swing and maximum at the centre twice per cycle, so the energy curve repeats **twice** per oscillation

- the period of the mass is therefore $2 \times 0.40\ \text{s} = 0.80\ \text{s}$

(b) $\omega = 2\pi/T$

$$\omega = \frac{2\pi}{0.80\ \text{s}} = \mathbf{7.9}\ \text{rad s}^{-1}$$

(c) the total energy equals the maximum kinetic energy, $E = \frac{1}{2}m\omega^2 x_0^2$

$$E = \frac{1}{2}(0.25\text{ kg})(7.85\text{ rad s}^{-1})^2(0.040\text{ m})^2 = \mathbf{1.2 \times 10^{-2}}\text{ J}$$

(d) $v = \omega\sqrt{x_0^2 - x^2}$

$$v = (7.85\text{ rad s}^{-1})\sqrt{(0.040\text{ m})^2 - (0.020\text{ m})^2} = (7.85\text{ rad s}^{-1})(0.0346\text{ m}) = \mathbf{0.27}\text{ m s}^{-1}$$

(e) light damping barely changes the period, so the time between maxima stays about 0.40 s

- the maxima get **lower** with time
- because the amplitude decays and the total energy is proportional to the **square** of the amplitude, so the energy falls faster than the amplitude does