

17.1 Simple harmonic oscillations

Name: _____ Class: _____ Date: _____

Total: 22 marks

Objective

Build the skills to answer exam questions on **simple harmonic motion** —the defining equation, the solutions, and the graphs.

You must be able to:

- use the SHM condition $a = -\omega^2 x$ and $x = x_0 \sin \omega t$
- use $v = \pm \omega \sqrt{x_0^2 - x^2}$ and $v_{\max} = \omega x_0$
- interpret the displacement, velocity and acceleration graphs

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

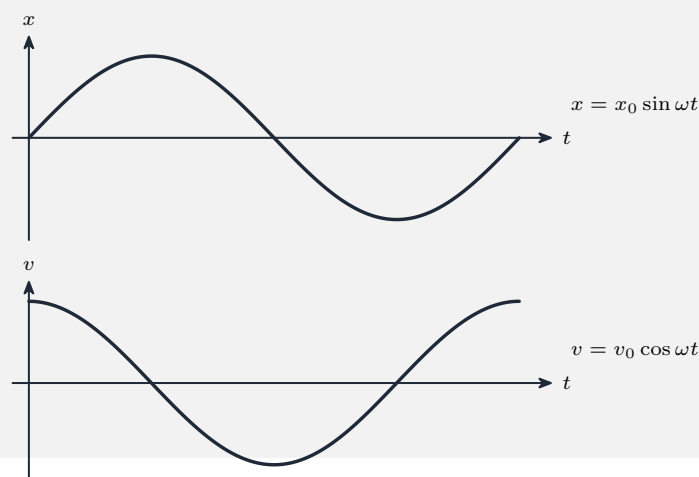
■ What SHM is

A particle moves with **SHM** when its acceleration is **proportional to displacement** and **directed back** to equilibrium:

$$a = -\omega^2 x, \quad x = x_0 \sin \omega t, \quad T = \frac{2\pi}{\omega}.$$

■ Velocity and the graphs

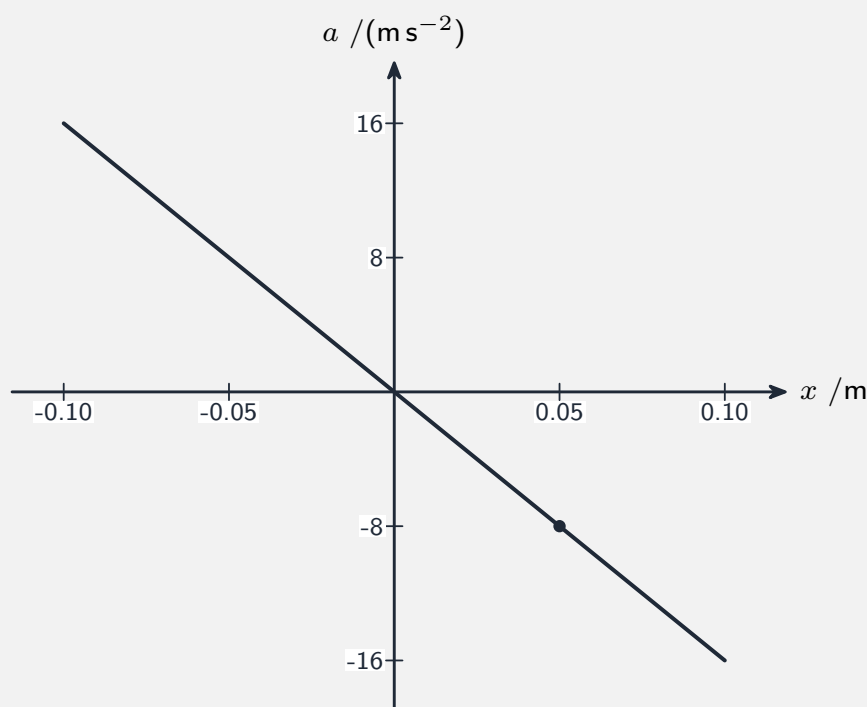
$$v = v_0 \cos \omega t, \quad v = \pm \omega \sqrt{x_0^2 - x^2}, \quad v_{\max} = \omega x_0, \quad a_{\max} = \omega^2 x_0.$$



Displacement and acceleration are **antiphase**; velocity leads displacement by a **quarter cycle**.

■ The acceleration–displacement graph

Because $a = -\omega^2 x$, a graph of a against x is a **straight line through the origin** with a **negative gradient** of magnitude ω^2 :



This is the **test for SHM**: $a \propto x$ (straight line through the origin) and **opposite in direction** (negative gradient). Read the gradient to find ω : gradient $= -\omega^2$, so $\omega = \sqrt{|\text{gradient}|}$.

Worked example. The line above passes through $(0.05 \text{ m}, -8 \text{ m s}^{-2})$, so gradient $= -8/0.05 = -160 \text{ s}^{-2} = -\omega^2$; thus $\omega = \sqrt{160} = 12.6 \text{ rad s}^{-1}$ and $T = 2\pi/\omega = 0.50 \text{ s}$.

2 Practice

Now apply the methods above.

2.1 State the **two** conditions for simple harmonic motion. [2]

2.2 Define the **amplitude** of an oscillation. [1]

2.3 A particle has $\omega = 5.0 \text{ rad s}^{-1}$ and amplitude 0.040 m . Find its **maximum speed**. [2]

2.4 State the **phase relationship** between displacement and acceleration in SHM. [1]

3 Exam-style questions

3.1 In simple harmonic motion, the acceleration is: [1]

- **A** proportional to displacement and in the same direction
 - **B** proportional to displacement and in the opposite direction
 - **C** constant
 - **D** always zero
-

3.2 A mass on a spring oscillates with period 0.50 s and amplitude 0.030 m . Find (a) the angular frequency, (b) the maximum speed, (c) the maximum acceleration. [4]

3.3 A particle in SHM has amplitude 0.10 m and $\omega = 8.0 \text{ rad s}^{-1}$. Find its **speed** when $x = 0.060 \text{ m}$. [3]

3.4 Sketch the **displacement–time** and **acceleration–time** graphs for SHM, and state

their phase relationship.

[3]

3.5 An object oscillates. A graph of its acceleration a against displacement x is a straight line through the origin passing through $(0.05 \text{ m}, -8.0 \text{ m s}^{-2})$.

(a) Explain how this graph shows the motion is **simple harmonic**.

[2]

(b) Use the **gradient** to find the angular frequency ω and the period T .

[3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **17.1 Simple harmonic oscillations** past-paper sheet in the Library, or try these in **Practice mode**:

- 9702/41 N25 —Q1 (SHM, structured)
- 9702/42 M25 —Q4 (a - x graph, ω)
- 9702/42 N25 —Q5 (SHM)
- 9702/43 J25 —Q5 (SHM calculation)
- 9702/42 J25 —Q5 (SHM graphs)

Solutions

2.1 Acceleration is **proportional to displacement** from a fixed point, and **directed towards** that point (opposite to displacement).

2.2 The **maximum displacement** from the equilibrium position.

2.3 $v_{\max} = \omega x_0 = 5.0 \times 0.040 = 0.20 \text{ m s}^{-1}$.

2.4 They are **antiphase** (phase difference $\pi / 180^\circ$).

3.1 B —proportional to displacement and opposite in direction.

3.2 (a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.50} = 12.6 \text{ rad s}^{-1}$. (b) $v_{\max} = \omega x_0 = 12.6 \times 0.030 = 0.38 \text{ m s}^{-1}$. (c) $a_{\max} = \omega^2 x_0 = 12.6^2 \times 0.030 = 4.7 \text{ m s}^{-2}$.

3.3 $v = \omega \sqrt{x_0^2 - x^2} = 8.0 \sqrt{0.10^2 - 0.060^2} = 8.0 \sqrt{0.0064} = 8.0 \times 0.080 = 0.64 \text{ m s}^{-1}$.

3.4 Displacement is a **sine** (or cosine) curve; acceleration is the **same shape inverted** (negative); they are **antiphase** (180° out of phase).

3.5 (a) The line is **straight and through the origin**, so a is **proportional to x** ; its **negative gradient** shows a is in the **opposite direction** to x —i.e. $a = -\omega^2 x$, the SHM condition.

3.5 (b) gradient $= \frac{-8.0}{0.05} = -160 \text{ s}^{-2} = -\omega^2$; $\omega = \sqrt{160} = 12.6 \text{ rad s}^{-1}$; $T = \frac{2\pi}{\omega} = 0.50 \text{ s}$.