A-LEVEL PHYSICS • EXERCISE SHEET

15.3 Kinetic theory of gases

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS root-mean-square 均方根 • kinetic theory 分子动理论 • kinetic energy 动能 • translational kinetic energy 平动动能 • mean-square speed 均方速率

Objective

Build the skills to answer exam questions on the **kinetic theory of gases**.

You must be able to:

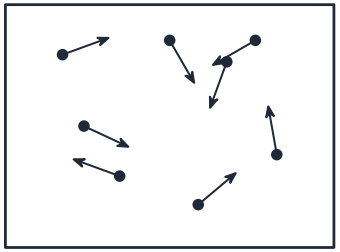
- state the **assumptions** of the kinetic theory and explain gas pressure
- use $pV = \frac{1}{3}Nm\langle c^2\rangle$ and the **r.m.s. speed**
- use mean translational KE $= \frac{3}{2}kT$

1 Worked examples

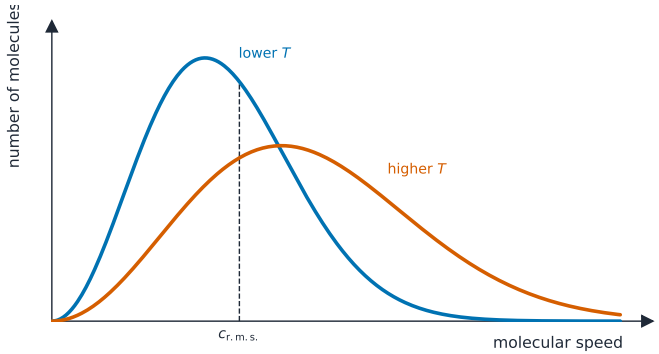
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kinetic model

A gas is many identical molecules in **continuous random motion**; their collisions with the walls cause **pressure**:



random motion — wall collisions cause pressure



higher T: peak → higher speeds, flatter curve

The Maxwell-Boltzmann speed distribution

Assumptions: many molecules, random motion; their volume is negligible; collision time is negligible; no forces except in collisions; collisions are **elastic**; Newton's laws hold.

■ Key results

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle, \qquad c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}.$$

Comparing with $pV = NkT$ gives the **mean translational KE** of a molecule:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT.$$

So molecular KE depends only on **temperature**.

2 Practice

Now apply the methods above.

2.1 State **two** assumptions of the kinetic theory of gases. [2]

2.2 Explain how gas molecules cause **pressure** on the walls of a container. [2]

2.3 State what is meant by the **root-mean-square (r.m.s.) speed**. [1]

2.4 Write the relationship $pV = \frac{1}{3}Nm\langle c^2\rangle$ in words. [1]

3 Exam-style questions

3.1 The **mean translational kinetic energy** of a gas molecule is: [1]

- **A** kT
- **B** $\frac{3}{2}kT$
- **C** $\frac{1}{2}mv^2$ only
- **D** RT

3.2 Calculate the mean translational kinetic energy of a gas molecule at 300 K. ($k = 1.38 \times 10^{-23}$.) [2]

3.3 A gas at 300 K has molecules of mass 5.3×10^{-26} kg. Calculate the **r.m.s. speed**. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 Explain, using the kinetic theory, why the pressure of a fixed mass of gas **increases** when it is heated at **constant volume**. [2]

3.5 A fixed sample of an ideal gas at thermodynamic temperature T has internal energy U.

(a) **State** what is meant by the **internal energy** of a system. [2]

(b) **Explain** why the internal energy of an **ideal** gas is directly proportional to its thermodynamic temperature. [2]

(c) The gas is first **compressed**, which raises its temperature to $3T$; work W is done on it during this compression. It is then **cooled at constant volume** back to $2T$. **Complete** the table, giving each entry in terms of some or all of W and U. [4]

process	**work done on the gas**	**thermal energy supplied to the gas**	**increase in internal energy**
compression to $3T$	W	_____	_____
cooling to $2T$	_____	_____	_____

(d) **Explain** why the work done on the gas during the second process is what it is. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- any two: many identical molecules in **random motion**; molecular **volume negligible**; **no forces** except during collisions; collisions are **elastic**; collision time negligible

2.2

- molecules collide with the walls and **change momentum** (rebound)
- this exerts a **force** on the wall; force over the wall area is the **pressure**

2.3

- the **square root of the mean of the squared speeds** of the molecules ($\sqrt{\langle c^2 \rangle}$)

2.4

- the product of pressure and volume equals **one third** of (the number of molecules × molecular mass × mean-square speed)

3.1 B

- mean kinetic energy of a molecule is $\frac{3}{2}kT$
- **A** is kT; **C** is a speed formula, not the mean KE; **D** is RT, which is for a mole, not one molecule

3.2

- $E = \frac{3}{2}kT$

$$E = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J K}^{-1})(300\ \text{K}) = \mathbf{6.2 \times 10^{-21}}\ \text{J}$$

3.3

- $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT \Rightarrow \langle c^2 \rangle = 3kT/m$

$$c_{\text{r.m.s.}} = \sqrt{\frac{3(1.38 \times 10^{-23}\ \text{J K}^{-1})(300\ \text{K})}{5.3 \times 10^{-26}\ \text{kg}}} = \mathbf{4.8 \times 10^{2}}\ \text{m s}^{-1}$$

3.4

- heating raises the **mean KE / mean speed** of the molecules
- they hit the walls **harder and more often**, so the pressure rises (volume fixed)

3.5

(a) the **sum of the random kinetic and potential energies** of all the molecules

- "random" means the motion has no overall direction — the whole container moving does not count

(b) in an ideal gas the molecules exert **no forces** on one another except during collisions, so the total **potential** energy is zero and the internal energy is entirely kinetic

- the mean kinetic energy of a molecule is $\frac{3}{2}kT$, i.e. proportional to T, so the total is proportional to T as well

(c) internal energy is proportional to T, so at T it is U, at $3T$ it is $3U$ and at $2T$ it is $2U$

process	**work done on the gas**	**thermal energy supplied**	**increase in internal energy**
compression to $3T$	W	$2U - W$	$+2U$
cooling to $2T$	0	$-U$	$-U$

- $+2U$; $-U$; the zero work; both thermal-energy entries, from $\Delta U = q + w$

(d) at **constant volume** the gas neither expands nor is compressed, so no work is done on or by it