A-LEVEL PHYSICS • EXERCISE SHEET

15.2 Equation of state

Name: ____________ Class: ________ Date: __________ **Total: 23 marks**

KEY WORDS ideal gas 理想气体 • boltzmann constant 玻尔兹曼常量 • equation of state 状态方程 • molar gas constant 摩尔气体常量 • pressure 压强

Objective

Build the skills to answer exam questions on the **equation of state** of an ideal gas.

You must be able to:

- use $pV = nRT$ and $pV = NkT$ (with T in kelvin)
- recall R, k and that $k = R/N_A$
- read a pV–T **graph**: gradient $= Nk$, so $N =$ gradient$/k$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Equation of state

An **ideal gas** obeys $pV \propto T$. Two equal forms:

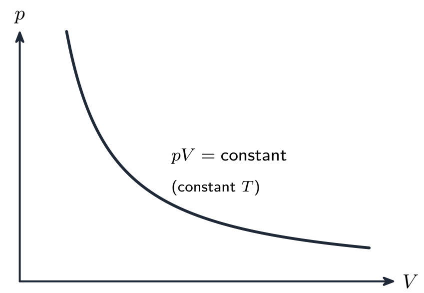
$$pV = nRT \quad (n = \text{moles}), \qquad pV = NkT \quad (N = \text{molecules}),$$

with $R = 8.31$ J mol^{-1} K^{-1}, $k = 1.38 \times 10^{-23}$ J K^{-1}, and $k = R/N_A$. **Always use Pa, m^3, K.**

Worked example. 0.50 mol at 300 K in 0.020 m^3: $p = \dfrac{nRT}{V} = \dfrac{0.50 \times 8.31 \times 300}{0.020} = 6.2 \times 10^4$ Pa.

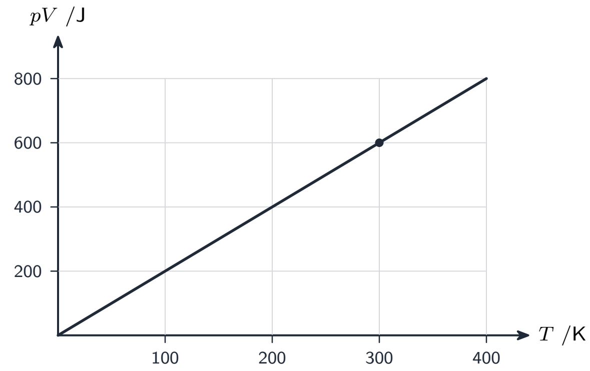
Boyle's law

At constant T (and amount), $p \propto 1/V$, so $pV =$ constant:



Reading a pV–T graph

For a fixed amount of gas, $pV = NkT$, so a graph of pV against T (in kelvin) is a **straight line through the origin** with **gradient** Nk:



A straight line **through the origin** confirms the gas is **ideal** ($pV \propto T$). To find the number of molecules, read the **gradient** and divide by k: $N = \dfrac{\text{gradient}}{k}$.

Worked example. The line above passes through (300 K, 600 J), so gradient $= 600/300 = 2.0$ J K^{-1} $= Nk$; thus $N = \dfrac{2.0}{1.38 \times 10^{-23}} = 1.4 \times 10^{23}$.

2 Practice

Now apply the methods above.

2.1 State what is meant by an **ideal gas**. [1]

2.2 Write the **two** forms of the ideal-gas equation of state. [2]

2.3 State the value of the **molar gas constant** R. [1]

2.4 A graph of pV against T for a fixed mass of ideal gas is a straight line through the origin. State what its **gradient** is equal to. [1]

3 Exam-style questions

3.1 In $pV = nRT$, the temperature T must be in: [1]

- **A** °C
- **B** K
- **C** J
- **D** mol

3.2 0.50 mol of an ideal gas at 300 K occupies 0.020 m^3. Calculate the **pressure**. ($R = 8.31$.) [3]

3.3 For a fixed mass of gas, the graph of pV against T is a straight line through the origin passing through $(300\text{ K}, 600\text{ J})$.

(a) State what this tells you about the gas. [1]

(b) Determine the number of molecules N in the sample. ($k = 1.38 \times 10^{-23}$.) [3]

3.4 State **Boyle's law**. [1]

3.5 For an ideal gas, two equations are used together:

$$pV = NkT \qquad \text{and} \qquad pV = \tfrac{1}{3}Nm\langle c^2\rangle$$

(a) **State** the meaning of the symbol T in the first equation, and **identify** the constant k. [2]

(b) **State** the meanings of the symbols m and $\langle c^2\rangle$ in the second equation. [2]

(c) Use the two equations to **derive** an expression, in terms of k and T only, for the mean kinetic energy E_K of one molecule of the gas. [3]

(d) Hence **state and explain** how the root-mean-square speed of the molecules changes when the thermodynamic temperature of the gas is **quadrupled**. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- a gas that obeys $pV \propto T$ (i.e. $pV = nRT$) exactly, at all p and T

2.2

- $pV = nRT$
- $pV = NkT$

2.3

- $R = 8.31\ \text{J mol}^{-1}\,\text{K}^{-1}$

2.4

- Nk (the number of molecules $\times$ the Boltzmann constant)

3.1 B

- T in $pV = nRT$ must be in **kelvin**
- **A** is Celsius; **C** is energy; **D** is amount of substance

3.2

- $p = nRT/V$

$$p = \frac{(0.50\ \text{mol})(8.31\ \text{J mol}^{-1}\,\text{K}^{-1})(300\ \text{K})}{0.020\ \text{m}^3} = \mathbf{6.2 \times 10^4}\ \text{Pa}$$

3.3

(a) it is an **ideal gas**: $pV \propto T$ (the line is straight and passes through the origin)

(b) gradient $= 600\ \text{J}/300\ \text{K} = 2.0\ \text{J K}^{-1}$

- this equals Nk

$$N = \frac{2.0\ \text{J K}^{-1}}{1.38 \times 10^{-23}\ \text{J K}^{-1}} = \mathbf{1.4 \times 10^{23}}$$

3.4

- at constant temperature (and amount of gas), the pressure is **inversely proportional** to the volume (pV = constant)

3.5

(a) T is the **thermodynamic (absolute) temperature**, measured in kelvin

- k is the **Boltzmann constant**

(b) m is the mass of **one molecule**

- $\langle c^2\rangle$ is the **mean square speed** — the average of the squares of the molecular speeds, not the square of the average

(c) set the two right-hand sides equal: $NkT = \frac{1}{3}Nm\langle c^2\rangle$

- so $\frac{1}{3}m\langle c^2\rangle = kT$
- the mean kinetic energy of one molecule is $E_{\mathrm{K}} = \frac{1}{2}m\langle c^2\rangle$
- which is $\frac{3}{2}$ times the previous line, so $E_{\mathrm{K}} = \frac{3}{2}kT$

(d) $\frac{1}{2}m\langle c^2\rangle \propto T$, so $\langle c^2\rangle \propto T$ and $c_{\mathrm{r.m.s.}} \propto \sqrt{T}$

- quadrupling T therefore **doubles** the r.m.s. speed