A-LEVEL PHYSICS • EXERCISE SHEET

# 13.4 Gravitational potential

Name: ____________ Class: ________ Date: __________ **Total: 16 marks**

**KEY WORDS** gravitational potential 引力势 • gravitational potential energy 重力势能 • mass 质量 • point mass 质点 • test mass 检验质量

## Objective

Build the skills to answer exam questions on **gravitational potential** — $\phi = -GM/r$ and gravitational potential energy.

**You must be able to:**

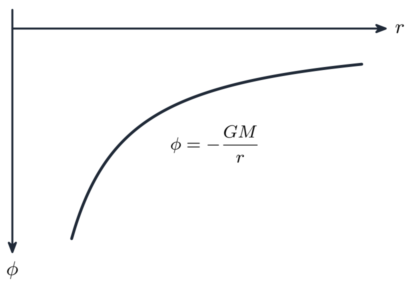
- define **gravitational potential** and use $\phi = -\dfrac{GM}{r}$
- use $E_P = -\dfrac{GMm}{r}$ for two point masses
- explain why potential and $E_P$ are negative

## 1 Worked examples

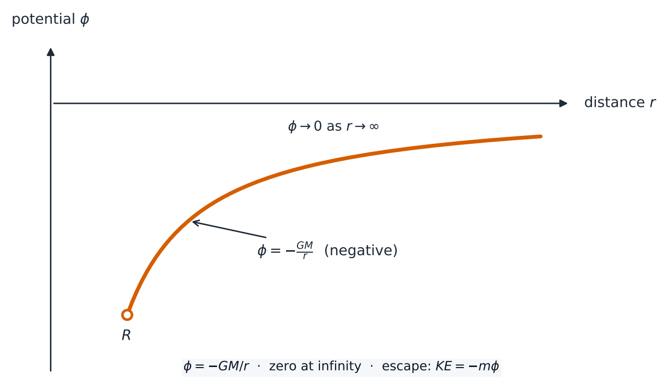
Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ Gravitational potential

**Gravitational potential** $\phi$ at a point is the **work done per unit mass** to bring a small test mass **from infinity** to that point. Potential is **zero at infinity** and **negative** everywhere else:






*The gravitational potential well*

$$\phi = -\frac{GM}{r} \quad (\mathrm{J\ kg^{-1}}), \qquad E_P = m\phi = -\frac{GMm}{r}.$$

For two radii, $\Delta E_P = GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$.

## 2 Practice

Now apply the methods above.

**2.1** Define **gravitational potential**. [1]

**2.2** State **why** gravitational potential is **negative**. [1]

**2.3** State the unit of gravitational potential. [1]

**2.4** Write the equation for the **gravitational potential energy** of two point masses. [1]

## 3 Exam-style questions

**3.1** The gravitational potential at **infinity** is: [1]

- **A** a maximum positive value
- **B** zero
- **C** a large negative value
- **D** infinite

**3.2** Calculate the gravitational potential at the Earth's surface. ($M = 6.0 \times 10^{24}$ kg, $R = 6.4 \times 10^{6}$ m, $G = 6.67 \times 10^{-11}$.) [3]

**3.3** Calculate the gravitational potential energy of a 1200 kg satellite at $r = 7.0 \times 10^{6}$ m from the Earth's centre. [2]

**3.4** Explain why **two closer** masses have **more negative** gravitational potential energy. [2]

**3.5** A 1000 kg satellite orbiting a planet is moved from an orbit of radius $7.0 \times 10^{6}$ m to one of radius $1.4 \times 10^{7}$ m. Its gravitational potential energy **increases** by $2.86 \times 10^{10}$ J. Calculate the **mass of the planet**, and give a **unit**. ($G = 6.67 \times 10^{-11}$.) [4]



## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- the **work done per unit mass** in bringing a small test mass from **infinity** to that point

**2.2**

- gravity is **attractive**, so it does the work as the mass moves in from infinity (where $\phi = 0$); the potential therefore becomes **negative**

**2.3**

- J $\text{kg}^{-1}$

**2.4**

- $E_\text{P} = -GMm/r$

**3.1 B**

- gravitational potential is defined as **zero** at infinity
- **A** would be a surface value; **C** is a typical Earth-surface $\phi$; **D** is not the definition

**3.2**

- $\phi = -GM/R$

$$\phi = -\frac{(6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(6.0 \times 10^{24}\ \text{kg})}{6.4 \times 10^{6}\ \text{m}} = \mathbf{-6.3 \times 10^{7}}\ \text{J kg}^{-1}$$

**3.3**

- $E_\text{P} = -GMm/r$

$$E_\text{P} = -\frac{(6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(6.0 \times 10^{24}\ \text{kg})(1200\ \text{kg})}{7.0 \times 10^{6}\ \text{m}} = \mathbf{-6.9 \times 10^{10}}\ \text{J}$$

**3.4**

- as $r$ decreases, $-GMm/r$ becomes **more negative**
- the masses are more **tightly bound** — more work would be needed to separate them to infinity

**3.5**

- $\Delta E_\text{P} = GMm(1/r_1 - 1/r_2)$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{7.0 \times 10^{6}\ \text{m}} - \frac{1}{1.4 \times 10^{7}\ \text{m}} = 7.14 \times 10^{-8}\ \text{m}^{-1}$$

$$M = \frac{2.86 \times 10^{10}\ \text{J}}{(6.67 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2})(1000\ \text{kg})(7.14 \times 10^{-8}\ \text{m}^{-1})} = \mathbf{6.0 \times 10^{24}}\ \text{kg}$$