A-LEVEL PHYSICS • EXERCISE SHEET

12.1 Kinematics of uniform circular motion

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS angular speed 角速度 • radian 弧度 • period 周期 • arc 弧 • revolution 圈

Objective

Build the skills to answer exam questions on **circular motion kinematics** — radians, angular speed, $v = r\omega$, and **connected** wheels.

You must be able to:

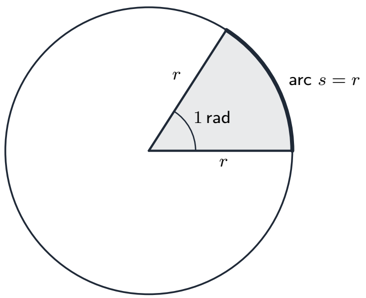
- define the **radian** and convert between radians and degrees
- use **angular speed** $\omega = \theta/t = 2\pi/T = 2\pi f$ and $v = r\omega$
- handle **connected wheels** (same axle $\rightarrow$ same ω; linked by a belt/chain $\rightarrow$ same rim speed)

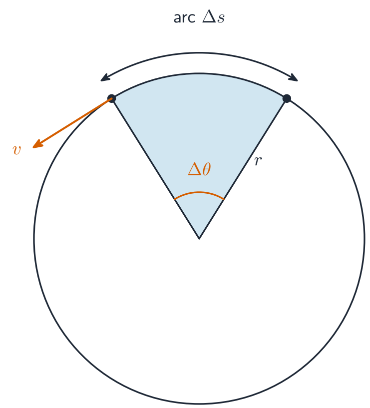
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Radians

One **radian** is the angle whose arc length equals the radius: $\theta = s/r$:





Angular speed in uniform circular motion

A full circle $= 2\pi$ rad; 1 rad $= 180°/\pi \approx 57.3°$. **Use radian mode** on your calculator.

■ Angular speed

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f \quad (\text{rad s}^{-1}), \qquad v = r\omega.$$

Worked example. A point on the equator (radius 6.4×10^6 m, period 24 h): $\omega = \dfrac{2\pi}{24 \times 3600} = 7.3 \times 10^{-5}$ rad s^{-1}; $v = r\omega = 465$ m s^{-1}.

■ Connected wheels

Exam questions often link wheels, cogs or pulleys. Two rules:

- **Fixed on the same axle** $\rightarrow$ they turn with the **same** ω (but different rim speeds if the radii differ).
- **Linked by a belt or chain** (no slipping) $\rightarrow$ the **rim speeds are equal**: $r_1\omega_1 = r_2\omega_2$.

Worked example. A belt links pulley A (radius 0.10 m, $\omega_A = 30$ rad s^{-1}) to pulley B (radius 0.050 m). Rim speed $v = r_A\omega_A = 0.10 \times 30 = 3.0$ m s^{-1}; so $\omega_B = \dfrac{v}{r_B} = \dfrac{3.0}{0.050} = 60$ rad s^{-1} (the smaller pulley spins faster).

2 Practice

Now apply the methods above.

2.1 Define the **radian**. [1]

2.2 Convert 90° to **radians**. [1]

2.3 A wheel turns at 2.0 revolutions per second. Find its **angular speed**. [2]

2.4 Two pulleys are linked by a belt. State what is **equal** for the two pulleys. [1]

3 Exam-style questions

3.1 A full circle, in radians, is: [1]

- **A** π
- **B** 2π
- **C** 360
- **D** 1

3.2 A car travels around a circular track of radius 50 m at 20 m s^{-1}. Find its **angular speed**. [2]

3.3 In a workshop, a motor turns a pulley of radius 25 mm at 1500 revolutions per minute. A tight belt runs from that pulley to a larger pulley of radius 75 mm on the shaft of a fan.

(a) Determine the angular speed of the **motor pulley**, in rad s^{-1}. [2]

(b) The belt does not slip. Determine the angular speed of the **fan**, and state what stays the same for both pulleys. [3]

3.4 Convert 1.5 rad to **degrees**. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the angle subtended at the centre of a circle by an arc whose **length equals the radius**

2.2

- a full circle is 2π rad $= 360°$

$$\theta = 90° \times \frac{2\pi \text{ rad}}{360°} = \frac{\pi}{\mathbf{2}} \text{ rad} \approx 1.57 \text{ rad}$$

2.3

- $f = 2.0$ rev s^{-1}; $\omega = 2\pi f$

$$\omega = 2\pi \times 2.0 \text{ s}^{-1} = 4\pi \text{ rad s}^{-1} = \mathbf{12.6} \text{ rad s}^{\mathbf{-1}}$$

2.4

- their **rim speeds** ($v = r\omega$) are equal (the belt does not slip)

3.1 B

- a full circle is 2π radians
- **A** is a half-turn; **C** is degrees; **D** is one radian, not a full turn

3.2

- $v = 20$ m s^{-1}, $r = 50$ m; $\omega = v/r$

$$\omega = \frac{20 \text{ m s}^{-1}}{50 \text{ m}} = \mathbf{0.40} \text{ rad s}^{\mathbf{-1}}$$

3.3

(a) 1500 revolutions per minute = 25 revolutions per second, and one revolution is 2π rad

$$\omega = 25 \text{ s}^{-1} \times 2\pi = \mathbf{157} \text{ rad s}^{\mathbf{-1}}$$

(b) the belt gives both pulleys the same **rim speed** $v = r\omega$

- motor radius 25 mm = 0.025 m

$$v = (0.025 \text{ m})(157 \text{ rad s}^{-1}) = 3.93 \text{ m s}^{-1}$$

$$\omega_{\text{fan}} = \frac{3.93 \text{ m s}^{-1}}{0.075 \text{ m}} = \mathbf{52} \text{ rad s}^{\mathbf{-1}}$$

- a third of the motor's, because its radius is three times larger

3.4

$$\theta = 1.5 \text{ rad} \times \frac{180°}{\pi} = \mathbf{86°}$$