A-LEVEL PHYSICS • EXERCISE SHEET

11.1.1 The alpha-scattering evidence, beta-plus decay and decay chains

Name: ____________ Class: ________ Date: __________ **Total: 33 marks**

KEY WORDS nucleus 原子核 • proton 质子 • isotope 同位素 • radioactive 放射性 • neutron 中子

Objective

Build the skills to answer exam questions on the nuclear atom that sheet 11.1 only starts: what each α**-scattering** result actually proves, how close an α-particle gets to a **nucleus** 原子核, the antiparticles and the **neutrinos** that go with β-decay, following a decay chain, and working in the **unified atomic mass unit** 统一原子质量单位. This sheet covers syllabus 11.1, learning outcomes 1 to 12.

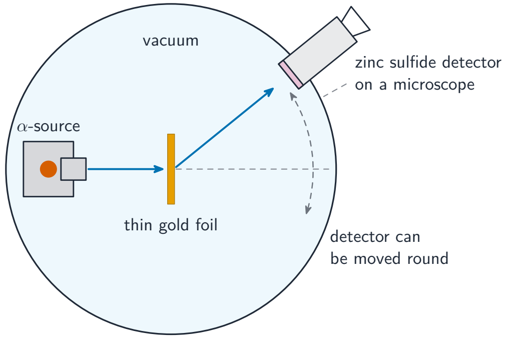
You must be able to:

- infer from the results of the α-particle scattering experiment the existence and small size of the nucleus
- understand and use the notation $^{A}_{Z}\mathrm{X}$ for the representation of **nuclides** 核素
- understand that nucleon number and charge are conserved in nuclear processes
- describe the composition, mass and charge of α-, β- and γ-radiations, including both β^- (electrons) and β^+ (positrons)
- understand that an **antiparticle** has the same mass but opposite charge to the corresponding particle, and that a positron is the antiparticle of an electron
- state that (electron) antineutrinos are produced during β^- decay and (electron) neutrinos are produced during β^+ decay
- understand that α-particles have discrete energies but that β-particles have a continuous range of energies
- represent α- and β-decay by a radioactive decay equation
- use the unified atomic mass unit (u) as a unit of mass

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ What each scattering result proves



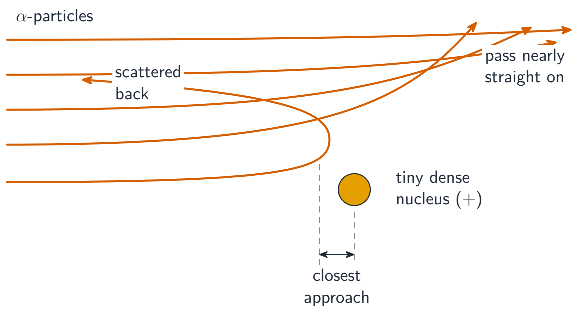
The mark scheme pairs each observation with one conclusion, so learn them as pairs.

Observation	Conclusion
almost all pass straight through	the atom is mostly empty space
a few are deflected through small angles	there is a concentrated **positive charge** that repels the α-particle
about 1 in 8000 is deflected through more than 90°	that charge sits in a very small, very massive nucleus

Three experimental details are asked just as often:

- the foil is **thin**, so each α-particle meets at most one nucleus,
- the chamber is **evacuated**, so air does not absorb or scatter the α-particles,
- **gold** is used because it can be beaten very thin and has a heavy, highly charged nucleus.

■ How close does an α-particle get?



An α-particle fired straight at a nucleus slows as it approaches, because its kinetic energy is being converted into electric potential energy. At the **closest approach** it is momentarily at rest, so all of its kinetic energy has become potential energy:

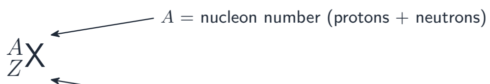
$$E_\mathrm{k} = \frac{Qq}{4\pi\varepsilon_0 d} = \frac{(Ze)(2e)}{4\pi\varepsilon_0 d}$$

An α-particle of kinetic energy 6.0 MeV is fired at a gold nucleus ($Z = 79$).

1. Convert the energy: $E_\mathrm{k} = (6.0 \times 10^6)(1.60 \times 10^{-19}) = 9.6 \times 10^{-13}$ J.
2. Rearrange: $d = \dfrac{2Ze^2}{4\pi\varepsilon_0 E_\mathrm{k}}$.
3. $d = \dfrac{2(79)(1.60 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})(9.6 \times 10^{-13})} = \mathbf{3.8 \times 10^{-14}}$ m.

- That is about 40 times the radius of the nucleus itself, so this α-particle never touches it. A faster α-particle gets closer: d is inversely proportional to E_k.

Nuclide notation and the charge-to-mass ratio



In ^A_ZX, Z is the **proton number** 质子数 and A the **nucleon number** 核子数, so the number of neutrons is $N = A - Z$. The nuclide is a nucleus of Z protons and N neutrons, with Z **orbital electrons** around it when the atom is neutral. Two nuclides with the same Z but a different A are **isotopes** 同位素 of one element: the same number of protons, a different number of neutrons.

A nucleus has 40 nucleons and a charge-to-mass ratio of 4.8×10^7 C kg^{-1}. How many protons has it?

1. Charge $= pe$, and mass $\approx Au = 40 \times 1.661 \times 10^{-27}$ kg.
2. $p = \dfrac{(4.8 \times 10^7)(40)(1.661 \times 10^{-27})}{1.60 \times 10^{-19}} = 19.9$.
3. A count of protons must be a whole number, so $p = \mathbf{20}$ and the nuclide is $^{40}_{20}$Ca.

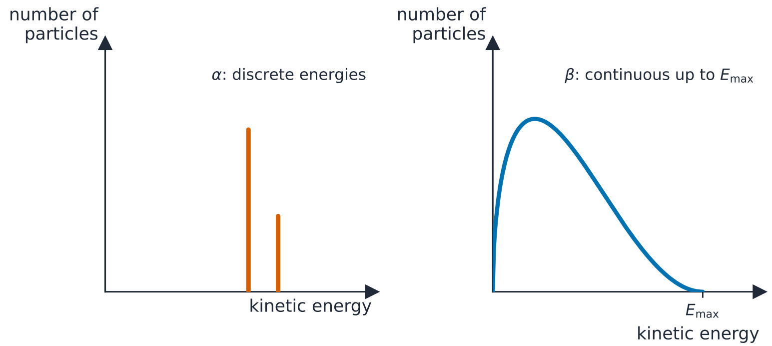
- Keep every figure until the end, then round. Rounding early can move the answer by a whole proton.

The radiations, their antiparticles and their neutrinos

	composition	charge	mass (u)	also emitted
α	2 protons + 2 neutrons	$+2e$	4	–
β^-	an electron	$-e$	0.0005	an **antineutrino**
β^+	a positron	$+e$	0.0005	a **neutrino**
γ	a photon	0	0	–

- An **antiparticle** has the **same mass** as its particle and the **opposite charge**. The positron is the antiparticle of the electron, and both are leptons.
- The pairing is examined directly: β^- goes with an **anti**neutrino, β^+ with a **neutrino**.

Why α energies are discrete but β energies are not

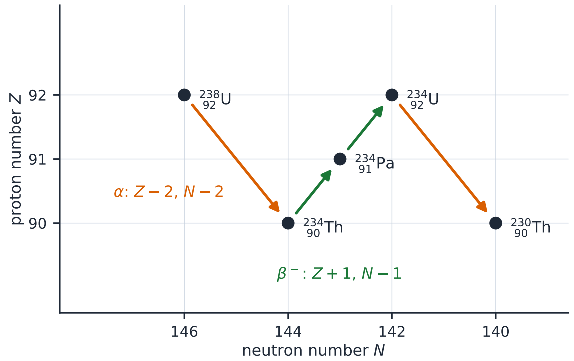


- In α-decay only two particles are produced, so the energy released is always shared the same way and every α-particle from that decay has the **same discrete** energy.
- In β-decay a third particle, the (anti)neutrino, is produced as well. It takes a varying share of the energy, so the β-particles come out with a **continuous range** up to a maximum.

■ **Decay equations and decay chains**

Balance the nucleon numbers along the top and the charges along the bottom; the (anti)neutrino is ${}^{0}_{0}$ and changes neither.

- β^+: ${}^{22}_{11}\text{Na} \to {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta + {}^{0}_{0}\nu$
- β^-: ${}^{90}_{38}\text{Sr} \to {}^{90}_{39}\text{Y} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$



On this grid every decay is the **same move every time**, which is what makes a long chain quick:

- α decay: $Z - 2$ and $N - 2$ (down two, left two),
- β^- decay: $Z + 1$ and $N - 1$ (up one, left one).

Working a chain backwards. ${}^{238}_{92}\text{U}$ decays in many steps to ${}^{206}_{82}\text{Pb}$. How many α- and β^--particles are emitted altogether?

1. Only α decay changes A, by 4 each time: $\dfrac{238 - 206}{4} = 8$ α-particles.
2. Those alone would take Z from 92 to $92 - 16 = 76$, but it must finish at 82.
3. Each β^- adds one to Z, so $82 - 76 = 6$ β^--particles. **8 α and 6 β^-.**

■ **The unified atomic mass unit**

$1\ \text{u} = 1.661 \times 10^{-27}$ kg, defined so that an atom of ${}^{12}_{6}\text{C}$ has a mass of exactly 12 u. To convert, multiply the mass in u by 1.661×10^{-27}: an α-particle is $4\ \text{u} = 6.64 \times 10^{-27}$ kg.

2 Practice

Now use the methods above.

2.1 In the α-scattering experiment, explain why the gold foil must be **thin** and why the chamber must be **evacuated**. [2]

2.2 An α-particle of kinetic energy 4.0 MeV is fired directly at a gold nucleus ($Z = 79$). Calculate its distance of closest approach. [3]

2.3 Write the complete decay equation for the β^+ decay of ${}^{22}_{11}\text{Na}$, including the third particle. [2]

2.4 State what is meant by an **antiparticle**, and name the antiparticle of the electron. [2]

2.5 Calculate the mass, in kg, of a nucleus of ${}^{40}_{20}\text{Ca}$. [2]

2.6 State the **two** quantities that must balance on each side of a nuclear decay equation. [2]

3 Exam-style questions

3.1 The mass of a certain nucleus is 3.99×10^{-26} kg. What is its nucleon number? [1]

A 12
B 16
C 24
D 40

3.2 A nucleus of $^{232}_{90}\text{Th}$ emits an α-particle, then a β^- particle, then a second α-particle. Determine the nucleon number and the proton number of the nuclide formed. [3]

3.3 An α-particle is fired directly at a gold nucleus.

(a) Describe the energy changes of the α-particle as it approaches the nucleus and comes momentarily to rest. [2]

(b) The α-particle has kinetic energy 5.0 MeV and the gold nucleus has 79 protons. Calculate the distance of closest approach. [3]

3.4 A nuclide X decays by β^- emission to a nuclide Y. Nuclide Y then decays by β^+ emission to a nuclide Z.

State how the proton number of Z compares with that of X, and name the particle emitted alongside the β particle in **each** of the two decays. [3]

3.5 The unified atomic mass unit is 1.661×10^{-27} kg and the elementary charge is 1.60×10^{-19} C.

(a) State what is meant by the **unified atomic mass unit**. [1]

(b) Calculate the mass, in kg, of an α-particle. [2]

(c) A different nucleus has 40 nucleons and a charge-to-mass ratio of 4.8×10^{7} C kg^{-1}. Determine its number of protons, and hence identify the nuclide. [3]

(d) Show that an α-particle has this same charge-to-mass ratio, and explain why. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- thin foil: so that each α-particle is scattered by **one nucleus only** (and is not absorbed)
- evacuated: so that the α-particles are not **absorbed or scattered by air**

2.2

- all the kinetic energy becomes electric potential energy at the closest approach: $E_k = \frac{2Ze^2}{4\pi\varepsilon_0 d}$
- $E_k = (4.0 \times 10^6)(1.60 \times 10^{-19}) = 6.4 \times 10^{-13}$ J

$$d = \frac{2(79)(1.60 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})(6.4 \times 10^{-13})} = \mathbf{5.7 \times 10^{-14}}\text{ m}$$

2.3

- nucleon number unchanged, proton number falls by one

$${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta + {}^{0}_{0}\nu$$

- the third particle is a (electron) **neutrino**, because this is β^+ decay

2.4

- an antiparticle has the **same mass** as the particle but the **opposite charge**
- the antiparticle of the electron is the **positron**

2.5

- mass $= A \times 1$ u $= 40 \times 1.661 \times 10^{-27}$

$$= \mathbf{6.6 \times 10^{-26}}\text{ kg}$$

2.6

- the **nucleon number** (the top numbers)
- the **charge**, i.e. the proton number (the bottom numbers)

3.1 C

- $A = \frac{3.99 \times 10^{-26}}{1.661 \times 10^{-27}} = 24.0$
- A and B are too light and D is nearly three times too heavy

3.2

- each α takes 4 from A and 2 from Z; each β^- leaves A alone and adds 1 to Z
- $A = 232 - 4 - 4 = \mathbf{224}$
- $Z = 90 - 2 + 1 - 2 = \mathbf{87}$

3.3

(a) the α-particle is **repelled** by the nucleus, so it slows down and its kinetic energy decreases

- that kinetic energy is converted into **electric potential energy**, and at the closest approach all of it has been converted

(b) $E_k = (5.0 \times 10^6)(1.60 \times 10^{-19}) = 8.0 \times 10^{-13}$ J

- $d = \frac{2Ze^2}{4\pi\varepsilon_0 E_k}$

$$d = \frac{2(79)(1.60 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})(8.0 \times 10^{-13})} = \mathbf{4.5 \times 10^{-14}}\text{ m}$$

3.4

- β^- decay raises the proton number by one and β^+ decay lowers it by one, so Z has the **same proton number as X**
- the β^- decay also emits an (electron) **antineutrino**
- the β^+ decay also emits an (electron) **neutrino**

3.5

(a) the unit of mass defined so that an atom of carbon-12 has a mass of exactly 12 u

(b) an α-particle has a mass of 4 u

$$m = 4 \times 1.661 \times 10^{-27} = \mathbf{6.6 \times 10^{-27}}\text{ kg}$$

(c) charge $= pe$ and mass $= 40 \times 1.661 \times 10^{-27} = 6.64 \times 10^{-26}$ kg

- $p = \frac{(4.8 \times 10^7)(6.64 \times 10^{-26})}{1.60 \times 10^{-19}} = 19.9$
- $p = \mathbf{20}$, so the nuclide is ${}^{40}_{20}\text{Ca}$

(d) an α-particle carries $2e$ and has a mass of 4 u:

$$\frac{2(1.60 \times 10^{-19})}{4(1.661 \times 10^{-27})} = \mathbf{4.8 \times 10^7}\text{ C kg}^{-1}$$

- both carry exactly **half an elementary charge per nucleon** (2 protons in 4 nucleons, 20 protons in 40), and the ratio depends only on that