A-LEVEL PHYSICS • EXERCISE SHEET

10.2 Kirchhoff's laws

Name: ______________ Class: __________ Date: ____________ **Total: 19 marks**

KEY WORDS kirchhoff's first law 基尔霍夫第一定律 • kirchhoff's second law 基尔霍夫第二定律 • resistance 电阻 • internal resistance 内阻 • parallel 并联

Objective

Build the skills to answer exam questions on **Kirchhoff's laws** and analysing **combined** circuits.

You must be able to:

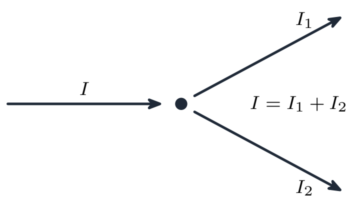
- state both **Kirchhoff's laws** and the conservation law each follows from
- combine resistors in **series** and **parallel**
- analyse a **combined** circuit: total resistance, total current, then each **branch** current and p.d.

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Kirchhoff's laws

First law (junction): current **in** = **current out** — from conservation of **charge**:



Second law (loop): around any closed loop, the **total e.m.f. = total p.d.** — from conservation of **energy**.

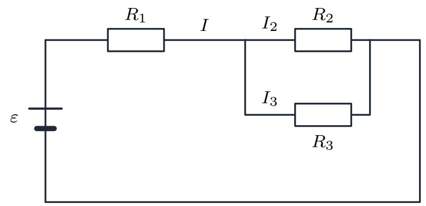
■ Combining resistors

- **Series** (same current): $R = R_1 + R_2 + \ldots$
- **Parallel** (same p.d.): $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$

Worked example. 3.0 Ω and 6.0 Ω in parallel: $\dfrac{1}{R} = \dfrac{1}{3.0} + \dfrac{1}{6.0} = \dfrac{1}{2.0}$, so $R = 2.0\ \Omega$.

■ Analysing a combined circuit

Real exam circuits mix series and parallel. Work in **steps**: collapse the parallel part, find the total current, then split it.



Worked example. $\varepsilon = 12$ V (negligible internal resistance), $R_1 = 4.0\ \Omega$ in series with $R_2 = 6.0\ \Omega$ and $R_3 = 3.0\ \Omega$ in parallel.

1. Parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = \dfrac{1}{2.0}$, so $R_\mathrm{p} = 2.0\ \Omega$.
2. Total: $R = R_1 + R_\mathrm{p} = 4.0 + 2.0 = 6.0\ \Omega$; current $I = \dfrac{\varepsilon}{R} = \dfrac{12}{6.0} = 2.0$ A.
3. P.d. across the parallel part: $V_\mathrm{p} = IR_\mathrm{p} = 2.0 \times 2.0 = 4.0$ V.
4. Branch currents: $I_2 = \dfrac{V_\mathrm{p}}{R_2} = \dfrac{4.0}{6.0} = 0.67$ A, $I_3 = \dfrac{V_\mathrm{p}}{R_3} = \dfrac{4.0}{3.0} = 1.33$ A (check: $0.67 + 1.33 = 2.0$ A ✓).

2 Practice

Now apply the methods above.

2.1 State **Kirchhoff's first law** and which conservation law it follows from. [2]

2.2 State **Kirchhoff's second law** and which conservation law it follows from. [2]

2.3 Find the combined resistance of 4.0 Ω and 6.0 Ω in **series**. [1]

2.4 Find the combined resistance of two 4.0 Ω resistors in **parallel**. [2]

3 Exam-style questions

3.1 Three 6.0 Ω resistors in **parallel** have a combined resistance of: [1]

- **A** 2.0 Ω
- **B** 6.0 Ω
- **C** 18 Ω
- **D** 0.50 Ω

3.2 A 12 V battery drives current through a 3.0 Ω and a 6.0 Ω resistor in **series**. Calculate the current and the p.d. across each resistor. [4]

3.3 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to $R_1 = 2.0\ \Omega$ in series with two resistors $R_2 = 4.0\ \Omega$ and $R_3 = 4.0\ \Omega$ in **parallel** (as in the worked example).

(a) Calculate the total resistance and the current from the battery. [3]

(b) Calculate the current in **each** of R_2 and R_3. [3]

3.4 At a junction, 5.0 A enters and one branch carries 2.0 A away. Find the current in the other branch. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **total current into a junction equals the total current out**
- it follows from conservation of **charge**

2.2

- around any closed loop, the **total e.m.f. equals the total p.d.**
- it follows from conservation of **energy**

2.3

- series: $R = R_1 + R_2$

$$R = 4.0\ \Omega + 6.0\ \Omega = \mathbf{10}$$

2.4

- parallel: $1/R = 1/R_1 + 1/R_2$

$$\frac{1}{R} = \frac{1}{4.0\ \Omega} + \frac{1}{4.0\ \Omega} = \frac{1}{2.0\ \Omega} \Rightarrow R = \mathbf{2.0}$$

3.1 A

- three equal resistors in parallel: $R = R_1/3$

$$R = \frac{6.0\ \Omega}{3} = \mathbf{2.0}$$

- **B** is one resistor; **C** is series; **D** would be twelve in parallel

3.2

- series: $R = 3.0\ \Omega + 6.0\ \Omega = 9.0\ \Omega$

$$I = \frac{12\ \text{V}}{9.0\ \Omega} = \mathbf{1.33}\ \text{A}$$

- $V = IR$

$$V_3 = (1.33\ \text{A})(3.0\ \Omega) = \mathbf{4.0}\ \text{V}, \qquad V_6 = (1.33\ \text{A})(6.0\ \Omega) = \mathbf{8.0}\ \text{V}$$

3.3

(a) parallel pair

$$\frac{1}{R_\text{p}} = \frac{1}{4.0\ \Omega} + \frac{1}{4.0\ \Omega} = \frac{1}{2.0\ \Omega} \Rightarrow R_\text{p} = 2.0\ \Omega$$

- total $R = R_1 + R_\text{p} = 2.0\ \Omega + 2.0\ \Omega = \mathbf{4.0}$

$$I = \frac{6.0\ \text{V}}{4.0\ \Omega} = \mathbf{1.5}\ \text{A}$$

(b) $V_\text{p} = IR_\text{p}$

$$V_\text{p} = (1.5\ \text{A})(2.0\ \Omega) = 3.0\ \text{V}$$

- each branch

$$I = \frac{3.0\ \text{V}}{4.0\ \Omega} = \mathbf{0.75}\ \text{A}$$

- the two currents are equal because $R_2 = R_3$

3.4

- Kirchhoff 1: current in = current out

$$I = 5.0\ \text{A} - 2.0\ \text{A} = \mathbf{3.0}\ \text{A}$$