A-LEVEL PHYSICS • EXERCISE SHEET

10.2.1 Kirchhoff's laws, deriving the resistor formulae and solving networks

Name: ______________ Class: __________ Date: ____________ **Total: 35 marks**

KEY WORDS kirchhoff's first law 基尔霍夫第一定律 • kirchhoff's second law 基尔霍夫第二定律 • conservation of energy 能量守恒 • conservation of charge 电荷守恒 • internal resistances 内阻

Objective

Build the skills to answer exam questions on **Kirchhoff's laws** that sheet 10.2 only begins: not just *stating* the two laws but **deriving** the series and parallel formulae from them, reducing a network one step at a time, deciding which component dissipates the most **power** 功率, and solving a circuit whose cells oppose each other. This sheet covers syllabus 10.2, learning outcomes 1 to 7.

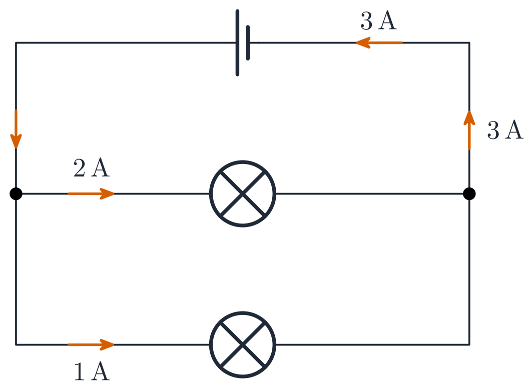
You must be able to:

- recall **Kirchhoff's first law** 基尔霍夫第一定律 and understand that it is a consequence of **conservation of charge** 电荷守恒
- recall **Kirchhoff's second law** 基尔霍夫第二定律 and understand that it is a consequence of **conservation of energy** 能量守恒
- derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in **series**
- use the formula for the combined resistance of two or more resistors in series
- derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in **parallel**
- use the formula for the combined resistance of two or more resistors in parallel
- use Kirchhoff's laws to solve simple circuit problems

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

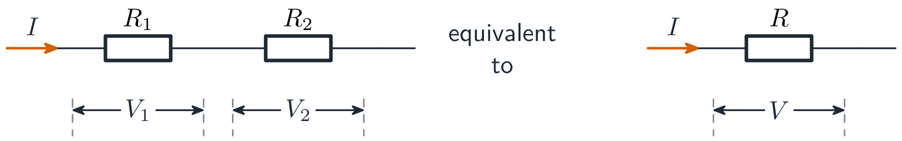
■ **The two laws, and what each one conserves**



at each junction: $3\,\mathrm{A} = 2\,\mathrm{A} + 1\,\mathrm{A}$

- **First law.** At any **junction** 节点, the total current in equals the total current out. Charge cannot pile up at a point, so this is conservation of charge.
- **Second law.** Round any closed loop, the total **electromotive force** 电动势 equals the total p.d. across the components. A unit of charge gives up exactly the energy it gained, so this is conservation of energy.
- The pairing is examined almost every session, and the two are easy to swap. First law $\to$ charge. Second law $\to$ energy.

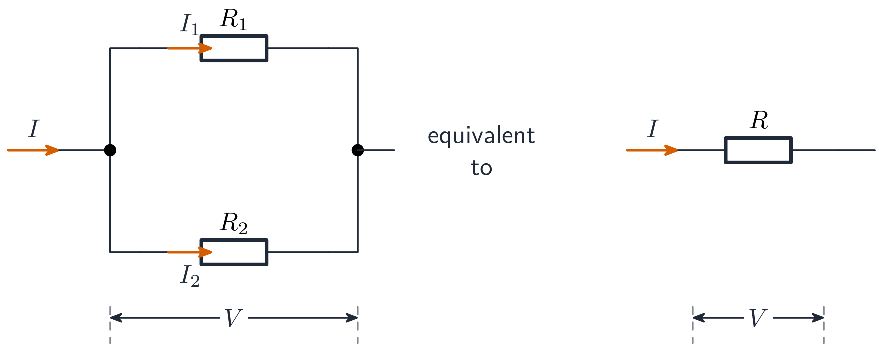
■ Deriving the series formula



A derivation must name the law at each step; that is what earns the marks.

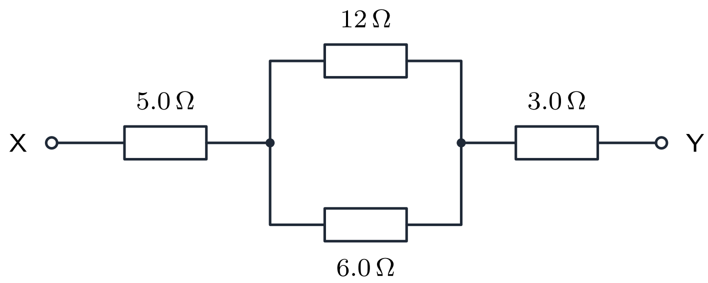
1. There is no junction between the resistors, so by the **first law** the same current I is in both.
2. By the **second law**, the p.d. across the combination is the sum of the p.d.s: $V = V_1 + V_2$.
3. Each p.d. is IR, so $IR = IR_1 + IR_2$.
4. Divide every term by I: $R = R_1 + R_2$.

■ Deriving the parallel formula



1. Each resistor forms its own loop with the source, so by the **second law** the same p.d. V is across both.
2. By the **first law** the current arriving at the junction is shared: $I = I_1 + I_2$.
3. Each current is V/R, so $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$.
4. Divide every term by V: $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

■ Reducing a network one step at a time



Never try to see the whole network at once. Replace one group, redraw, repeat.

1. The parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{12} + \dfrac{1}{6.0} = \dfrac{1}{12} + \dfrac{2}{12} = \dfrac{3}{12}$, so $R_\mathrm{p} = 4.0\ \Omega$.
2. What is left is three resistances in series: $5.0 + 4.0 + 3.0$.
3. $R_{XY} = \mathbf{12}\ \Omega$.

- For **two** resistors only, $R_\mathrm{p} = \dfrac{R_1 R_2}{R_1 + R_2}$ is quicker. It is wrong for three.
- A parallel combination is always **smaller** than its smallest resistor; a series one is always larger than its largest. Use that to check every answer.

■ Which component dissipates the most power

Two rules settle this with no arithmetic at all, and which one applies depends on what the components share.

- **Same current** (in series): $P = I^2 R$, so the **larger** resistance dissipates more.

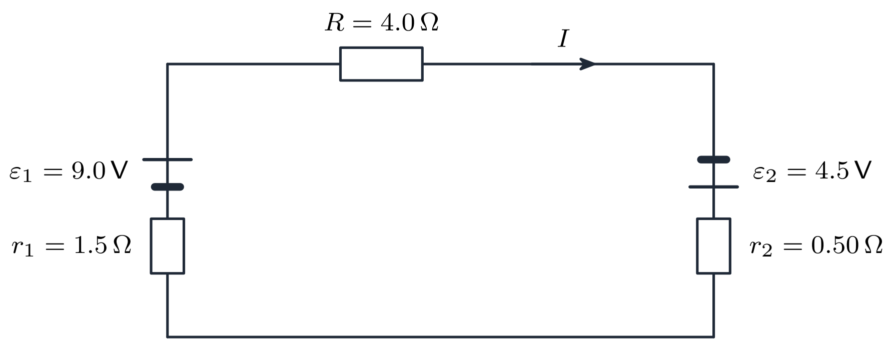
- **Same p.d.** (in parallel): $P = \dfrac{V^2}{R}$, so the **smaller** resistance dissipates more.

A 4.0 Ω and an 8.0 Ω resistor are joined to a 12 V supply of negligible internal resistance.

1. **In series:** $R = 12\ \Omega$, so $I = 1.0$ A in both. $P_4 = (1.0)^2(4.0) = 4.0$ W and $P_8 = (1.0)^2(8.0) = 8.0$ W – the larger resistor wins.
2. **In parallel:** both have 12 V across them. $P_4 = \dfrac{12^2}{4.0} = 36$ W and $P_8 = \dfrac{12^2}{8.0} = 18$ W – now the smaller resistor wins.

- In a mixed circuit, the resistor carrying the **whole** current usually dissipates the most, because every parallel branch carries only a share of it.

■ A loop with two cells that oppose



The second law handles this with no extra ideas: go once round the loop and add the e.m.f.s with their signs.

1. The cells are connected the opposite way round, so the net e.m.f. is $9.0 - 4.5 = 4.5$ V.
2. The two **internal resistances** 内阻 are in series with the resistor: $R_{\text{total}} = 1.5 + 0.50 + 4.0 = 6.0\ \Omega$.
3. $I = \dfrac{4.5}{6.0} = \mathbf{0.75}$ A, driven the way the larger e.m.f. pushes.
4. The terminal p.d. of the 9.0 V cell is $\varepsilon - Ir = 9.0 - (0.75)(1.5) = \mathbf{7.9}$ V.

- The smaller cell is being driven backwards, so it is charging; its terminal p.d. is $4.5 + (0.75)(0.50)$, which is *larger* than its e.m.f.

2 Practice

Now use the methods above.

2.1 Calculate the combined resistance of 2.0 Ω, 3.0 Ω and 6.0 Ω connected in parallel. [2]

2.2 For the network between X and Y in the worked example above, the 12 Ω resistor is replaced by a second 6.0 Ω resistor. Calculate the new resistance between X and Y. [3]

2.3 Explain why connecting a second resistor in parallel with the first always **decreases** the resistance between the same two points. [2]

2.4 A 4.0 Ω and an 8.0 Ω resistor are connected in **series** across a supply. State which dissipates the greater power, and by what factor. [2]

2.5 The same two resistors are now connected in **parallel** across the same supply. State which dissipates the greater power, and explain why the answer has changed. [2]

3 Exam-style questions

3.1 A 6.0 Ω resistor and a 3.0 Ω resistor are connected in parallel. This combination is in series with a 4.0 Ω resistor across a supply of e.m.f. 12 V and negligible internal resistance. What is the current from the supply? [1]

A	B
C	D

A 0.50 A
B 1.0 A
C 2.0 A
D 3.0 A

3.2 Two resistors of resistances R_1 and R_2 are connected in parallel. Use Kirchhoff's laws to show that the combined resistance R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Name the law used at each step. [4]

3.3 A battery of e.m.f. 18 V and negligible internal resistance is connected to a 6.0 Ω resistor in series with a parallel combination of a 9.0 Ω resistor and an 18 Ω resistor.

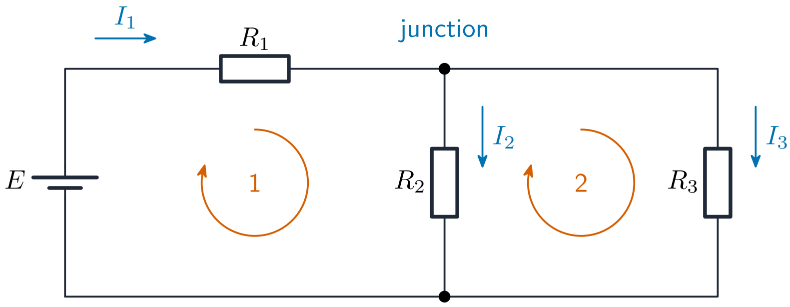
(a) Show that the resistance of the parallel combination is 6.0 Ω. [2]

(b) Calculate the current in the battery. [2]

(c) Calculate the p.d. across the parallel combination and the current in each of its branches. [3]

(d) State, with a reason, which of the three resistors dissipates the greatest power. [2]

3.4 A cell of e.m.f. 12 V and negligible internal resistance is connected to $R_1 = 3.0\ \Omega$ in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 4.0\ \Omega$, as shown.



(a) Write down the equation that Kirchhoff's first law gives for the junction. [1]

(b) Calculate the current I_1 in the cell and the currents I_2 and I_3 in the two branches. [4]

(c) Calculate the p.d. across R_1. [2]

(d) Show that the total power dissipated in the three resistors equals the power supplied by the cell, and state which conservation law this illustrates. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$$\frac{1}{R} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = \frac{3}{6.0} + \frac{2}{6.0} + \frac{1}{6.0} = 1.0 \Rightarrow R = \mathbf{1.0}\ \Omega$$

- the answer is smaller than the smallest resistor, as it must be

2.2

- the parallel pair is now two equal resistors, so $R_\mathrm{p} = \dfrac{6.0}{2} = 3.0\ \Omega$
- in series with the others: $R_{XY} = 5.0 + 3.0 + 3.0 = \mathbf{11}\ \Omega$

2.3

- the second resistor gives the charge an extra path, so for the same p.d. the total current is larger
- $R = V/I$, so a larger current for the same p.d. means a smaller resistance

2.4

- in series the current is the same in both, and $P = I^2R$
- the **8.0** Ω resistor, dissipating **twice** the power of the 4.0 Ω

2.5

- in parallel the p.d. is the same across both, and $P = V^2/R$, so the **4.0** Ω resistor dissipates the greater power
- the answer changes because the resistors now share the p.d. instead of the current

3.1 C

- parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = \dfrac{1}{2.0}$, so $R_\mathrm{p} = 2.0\ \Omega$
- total $= 4.0 + 2.0 = 6.0\ \Omega$, so $I = \dfrac{12}{6.0} = 2.0$ A

3.2

- by **Kirchhoff's second law** each resistor is in its own loop with the source, so the same p.d. V is across both
- by **Kirchhoff's first law** the currents at the junction add: $I = I_1 + I_2$
- substitute $I = V/R$ for each: $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$, so $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$
- put the right-hand side over a common denominator and invert:

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

3.3

(a) $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{9.0} + \dfrac{1}{18} = \dfrac{2}{18} + \dfrac{1}{18} = \dfrac{3}{18}$

- $R_\mathrm{p} = \dfrac{18}{3} = \mathbf{6.0}\ \Omega$

(b) total resistance $= 6.0 + 6.0 = 12\ \Omega$

$$I = \frac{18}{12} = \mathbf{1.5}\ \mathrm{A}$$

(c) $V_\mathrm{p} = IR_\mathrm{p} = (1.5)(6.0) = \mathbf{9.0}\ \mathrm{V}$

- $I_9 = \dfrac{9.0}{9.0} = \mathbf{1.0}$ A and $I_{18} = \dfrac{9.0}{18} = \mathbf{0.50}$ A
- check: $1.0 + 0.50 = 1.5$ A, the current in the battery

(d) the **6.0** Ω resistor

- it carries the whole 1.5 A while each branch carries only part of it, and $P = I^2R$: 13.5 W against 9.0 W and 4.5 W

3.4

(a) $I_1 = I_2 + I_3$

(b) parallel pair: $\dfrac{1}{R_\mathrm{p}} = \dfrac{1}{12} + \dfrac{1}{4.0} = \dfrac{1}{12} + \dfrac{3}{12} = \dfrac{4}{12}$, so $R_\mathrm{p} = 3.0\ \Omega$

- total $= 3.0 + 3.0 = 6.0\ \Omega$, so $I_1 = \dfrac{12}{6.0} = \mathbf{2.0}$ A
- p.d. across the pair $= (2.0)(3.0) = 6.0$ V
- $I_2 = \dfrac{6.0}{12} = \mathbf{0.50}$ A and $I_3 = \dfrac{6.0}{4.0} = \mathbf{1.5}$ A

(c) $V_1 = I_1R_1$

$$V_1 = (2.0)(3.0) = \mathbf{6.0}\ \mathrm{V}$$

(d) $P = I^2R$ for each resistor

- $P_1 = (2.0)^2(3.0) = 12$ W, $P_2 = (0.50)^2(12) = 3.0$ W, $P_3 = (1.5)^2(4.0) = 9.0$ W, total $=$ **24** W
- the cell supplies $\varepsilon I_1 = (12)(2.0) = 24$ W, the same, illustrating **conservation of energy** – which is what Kirchhoff's second law expresses