A-LEVEL PHYSICS • EXERCISE SHEET

# 10.1.1 e.m.f., internal resistance and the V-I graph

Name: ____________ Class: ________ Date: __________ **Total: 30 marks**

**KEY WORDS** resistance 电阻 • internal resistance 内阻 • potential difference 电势差 • terminal p.d. 端电压 • circuit symbols 电路符号

## Objective

Sheet 10.1 is the thinnest sheet in this subject – four one-mark definitions – against a unit that the exam asks eleven-mark questions on. This sheet is the rest of 10.1: reading and **drawing circuit diagrams** 电路图 with the standard symbols, the **energy** definitions of **e.m.f.** 电动势 and **potential difference** 电势差 and the difference between them, what the **internal resistance** 内阻 of a source does to the **terminal p.d.** 端电压, and the $V$–$I$ experiment that measures both at once. This sheet covers syllabus 10.1.

**You must be able to:**

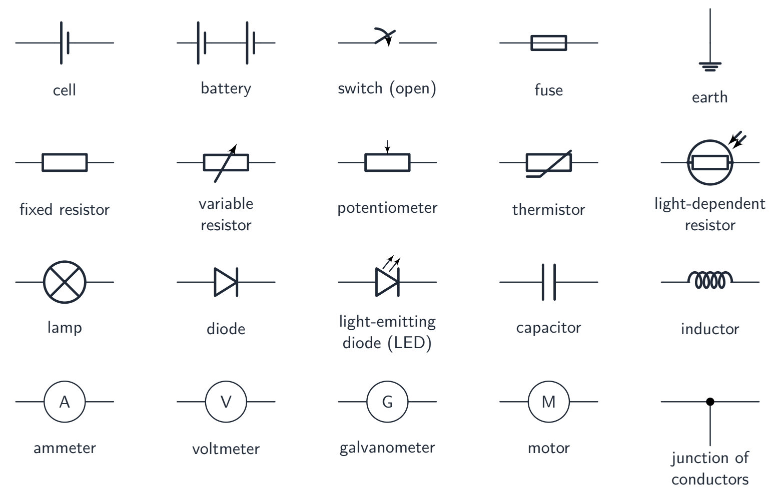
- recall, draw and interpret the standard **circuit symbols** 电路符号 and circuit diagrams
- define the e.m.f. of a source as the **energy transferred per unit charge** in driving charge round a complete circuit
- distinguish between e.m.f. and potential difference in terms of energy
- understand the effect of the internal resistance of a source on the terminal potential difference

## 1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

### ■ The symbols, and how a diagram is marked





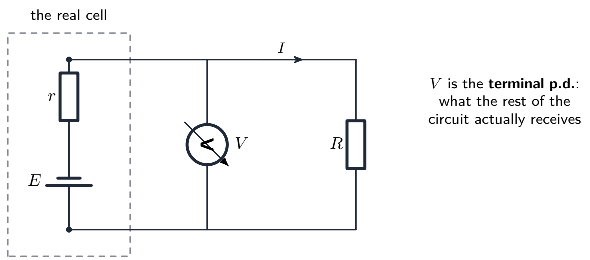

- Draw with a **ruler**, with straight wires and square corners; a wire that simply ends is a lost mark.
- An **ammeter** goes **in series** with the component whose current you want; a **voltmeter** goes **in parallel** across the component whose p.d. you want. Swapping them is the commonest diagram error.
- Junctions that are connected get a **dot**; wires that cross without connecting do not.

### ■ Two definitions, and they differ only in direction

| | definition | where the energy goes |
|---|---|---|
| **electromotive force (e.m.f.)** $E$ | the energy transferred **to** each unit of charge, from some other form into electrical, in driving it around a complete circuit | chemical → electrical, inside the source |
| **p.d.** $V$ | the energy transferred **from** each unit of charge into another form, in a component | electrical → thermal, light, kinetic, in the component |

- Both are **joules per coulomb**, so both are measured in **volts**. That is why the distinction has to be made in words about **energy**, and why "e.m.f. is the voltage of the battery" earns nothing.
- E.m.f. is **not a force**, despite its name.

## ■ A real cell: an ideal source with a resistance in series




Each coulomb is given $E$ joules by the source, gives up $Ir$ joules inside the cell and $V$ joules outside it, so $E = V + Ir$, or $V = E - Ir$. Nothing is lost – the missing energy is dissipated *inside* the cell, which is why a battery gets warm.

Follow one coulomb round the circuit:

$$E = V + Ir \quad \Longleftrightarrow \quad V = E - Ir \quad \Longleftrightarrow \quad E = I(R + r)$$

- It is given $E$ joules by the source.
- It gives up $Ir$ joules **inside** the cell, warming it – these are sometimes called the ”lost volts”.
- It gives up $V$ joules **outside**, in the rest of the circuit. $V$ is the **terminal p.d.**: what the circuit actually receives.

Two limiting cases the exam likes:

| situation | what happens |
|---|---|
| **no current drawn** ($I = 0$) | $V = E$: the terminal p.d. **equals** the e.m.f., because nothing is lost inside |
| **terminals shorted** ($R = 0$) | $I = \frac{E}{r}$, the **largest current** the cell can deliver, and all of the energy is wasted inside it |

*Worked example. A cell of e.m.f.* 1.5 V *and internal resistance* 0.50 Ω *is connected to a* 2.5 Ω *resistor. Find the current and the terminal p.d.*

- the e.m.f. drives the current through **both** resistances

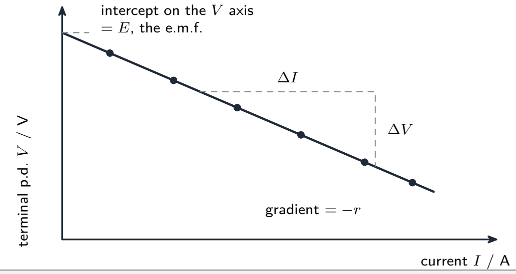
$$I = \frac{E}{R + r} = \frac{1.5\ \text{V}}{2.5\ \Omega + 0.50\ \Omega} = 0.50\ \text{A}$$

- then the terminal p.d. is what is left after the internal drop

$$V = E - Ir = 1.5\ \text{V} - 0.50\ \text{A} \times 0.50\ \Omega = \mathbf{1.25}\ \text{V}$$



## ■ The experiment: one graph, two answers




Vary the external resistance, plot $V$ against $I$, and $V = E - Ir$ gives a straight line: the **intercept** is the e.m.f. and the **gradient** is minus the internal resistance. Extrapolating back to $I = 0$ is how you measure an e.m.f. you can never read directly, because any real voltmeter draws some current.

Vary the external resistance, record $V$ and $I$ each time, and plot $V$ against $I$. Since

$$V = E - rI$$

is $y = c + mx$:

- the **intercept** on the $V$ axis is the **e.m.f.** $E$;
- the **gradient** is $-r$, so the internal resistance is **minus** the gradient, and it is positive.
- This is how an e.m.f. is measured, because a real voltmeter always draws a **small current**, so the reading straight across the cell is always slightly **less** than $E$. The graph extrapolates to the $I = 0$ case that cannot quite be reached.
- **Power:** the source supplies $EI$ in total, of which $I^2r$ is wasted inside and $VI$ is delivered outside.

# 2 Practice

Now use the methods above.

**2.1** Define the **e.m.f.** of a source. [2]

**2.2** Explain, in terms of energy, the difference between the e.m.f. of a cell and the potential difference across a resistor. [2]

**2.3** A cell of e.m.f. 1.5 V and internal resistance 0.40 Ω delivers a current of 0.60 A. Calculate the terminal p.d. [2]

**2.4** Explain why the terminal p.d. of a cell falls as the current drawn from it increases. [2]

**2.5** State the terminal p.d. of a cell when **no** current is drawn from it, and explain your answer. [2]

**2.6** Calculate the largest current the cell in **2.3** could deliver, and state the condition under which this happens. [2]

## 3 Exam-style questions

**3.1** As the current drawn from a cell increases, the terminal potential difference [1]

A B C D

**A** increases, because more energy is transferred each second.
**B** stays the same, because the e.m.f. of the cell is fixed.
**C** decreases, because more energy is dissipated in the internal resistance.
**D** decreases, because the e.m.f. of the cell falls.



**3.2** A battery of e.m.f. 12 V and internal resistance 0.50 Ω is connected to a 2.5 Ω resistor in series with a parallel combination of a 6.0 Ω and a 3.0 Ω resistor.

(a) Draw a circuit diagram of this arrangement, including an ammeter to measure the current from the battery and a voltmeter to measure the terminal p.d. [2]

(b) Calculate the total resistance of the external circuit. [2]

(c) Calculate the current delivered by the battery. [2]

**3.3** A student varied the resistance of the external circuit and recorded the terminal p.d. $V$ and the current $I$.

| $I$ / A | **0.20** | **0.45** | **0.70** | **0.95** | **1.20** |
|---|---|---|---|---|---|
| $V$ / V | 5.72 | 5.45 | 5.18 | 4.91 | 4.64 |

(a) State the equation relating $V$ and $I$ for this circuit, and state what the gradient and the intercept of a graph of $V$ against $I$ represent. [3]

(b) Use the first and last pairs of readings to calculate the internal resistance of the source. [3]

(c) Calculate the e.m.f. of the source. [2]

(d) Calculate the power dissipated inside the source when the current is 1.20 A. [2]

(e) Explain why connecting a voltmeter directly across the terminals of the source does not give exactly its e.m.f. [1]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- the **energy transferred to each unit of charge** from another form into electrical form
- in driving that charge **around a complete circuit**

**2.2**

- the **e.m.f.** is the energy given **to** each coulomb by the source, as chemical energy becomes electrical
- the **p.d.** is the energy given **up** by each coulomb in the component, as electrical energy becomes thermal or another form

**2.3**

- use $V = E - Ir$

$$V = 1.5\ \text{V} - 0.60\ \text{A} \times 0.40\ \Omega = \mathbf{1.26}\ \text{V}$$

**2.4**

- a larger current means a larger p.d. $Ir$ across the **internal resistance**
- the e.m.f. is fixed, so with more of it used inside the cell, less is left for the external circuit and $V = E - Ir$ falls

**2.5**

- the terminal p.d. **equals the e.m.f.**
- with $I = 0$ there is no p.d. across the internal resistance, so nothing is lost inside the cell

**2.6**

- the largest current flows when the external resistance is **zero** – the terminals are short-circuited

$$I_{\text{max}} = \frac{E}{r} = \frac{1.5\ \text{V}}{0.40\ \Omega} = \mathbf{3.75}\ \text{A}$$

**3.1 C**

- the internal resistance is unchanged, but a larger current gives a larger $Ir$ inside the cell, so less of the fixed e.m.f. is left as terminal p.d.
- **D** is the standard error: the e.m.f. does **not** change with current

**3.2**

(a)

- the battery, the 2.5 Ω resistor and the parallel pair correctly in **series**, with the 6.0 Ω and 3.0 Ω in **parallel** with each other
- the **ammeter in series** with the battery and the **voltmeter in parallel** across its terminals, with correct symbols



(b)

- combine the parallel pair first

$$\frac{1}{R_\text{p}} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{1}{2.0\ \Omega}, \qquad R_\text{p} = 2.0\ \Omega$$

$$R_{\text{ext}} = 2.5\ \Omega + 2.0\ \Omega = \mathbf{4.5}$$

(c)

- the e.m.f. drives the current through the external resistance **and** the internal resistance

$$I = \frac{E}{R_{\text{ext}} + r} = \frac{12\ \text{V}}{4.5\ \Omega + 0.50\ \Omega} = \mathbf{2.4}\ \text{A}$$

**3.3**

(a)

- $V = E - Ir$
- the **intercept** on the $V$ axis is the **e.m.f.** $E$
- the **gradient** is $-r$, so the internal resistance is minus the gradient

(b)

- the internal resistance is minus the gradient of $V$ against $I$

$$\text{gradient} = \frac{4.64\ \text{V} - 5.72\ \text{V}}{1.20\ \text{A} - 0.20\ \text{A}} = -1.08\ \Omega$$

$$r = \mathbf{1.08}$$

(c)

- substitute one pair of readings into $E = V + Ir$

$$E = 5.72\ \text{V} + 0.20\ \text{A} \times 1.08\ \Omega = \mathbf{5.94}\ \text{V}$$

(d)

- the power dissipated inside the source is $I^2 r$

$$P = (1.20\ \text{A})^2 \times 1.08\ \Omega = \mathbf{1.6}\ \text{W}$$

(e)

- a real voltmeter draws a small current, so there is a small p.d. $Ir$ inside the cell and the reading is slightly **less** than the e.m.f.